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augmenter la motivation, l'auto-régulation et la réussite scolaire des  
étudiants à l'aide de l'apprentissage par l'enseignement**

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# **Learning by Teaching: Fostering Self-Regulatory Strategies and Achievement during Complex Mathematics Problem Solving**

1. Muis, K. R., Psaradellis, C., Chevrier, M., Di Leo, I., & Lajoie, S. P. (2016). Learning by preparing to teach: Fostering self-regulatory processes and achievement during complex mathematics problem solving. *Journal of Educational Psychology*, 108, 474-492.

Includes full literature review and details on the methodology and results for Study 1.

2. Muis, K. R., Psaradellis, C., Lajoie, S. P., Di Leo, I., & Chevrier, M. (2015). The role of epistemic emotions in mathematics problem solving. *Contemporary Educational Psychology*, 42, 172-185.

For this publication, we focused on how emotions affect mathematics problem solving. The same dataset was used, but the focus was on emotions (given no differences between groups on emotions in the main study, reported above, the groups were merged).

3. The images are examples of the Teachable Agent App that we developed. The last two pages are data that we extracted from the log files from an actual student participant (Year 3 data).

# Learning by Preparing to Teach: Fostering Self-Regulatory Processes and Achievement During Complex Mathematics Problem Solving

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We developed an intervention based on the learning by teaching paradigm to foster self-regulatory processes and better learning outcomes during complex mathematics problem solving in a technology-rich learning environment. Seventy-eight elementary students were randomly assigned to 1 of 2 conditions: learning by preparing to teach, or learning for learning (control condition). Students' conceptualizations (task definitions) of the problem, self-regulatory processes, and mathematics achievement were then compared across the 2 conditions. To measure task definitions of the mathematics problem, students developed concept maps of the problem using a tablet application. To capture self-regulatory processes, students were asked to think out loud as they solved the problem. Results revealed that students in the learning by preparing to teach intervention developed a more detailed and better-organized concept map of the problem compared with students in the control condition. Students in the learning by preparing to teach intervention also engaged in more metacognitive processing strategies and had higher levels of mathematics problem solving achievement compared with students in the control condition. No differences were found, however, in planning and goal setting or in use of cognitive strategies across the 2 conditions. Implications of this research suggest students' initial task definitions may be a key factor in differences found when learning by teaching compared with solely learning for learning.

**Keywords:** concept maps, self-regulatory processes, mathematics achievement, learning by teaching

When it comes to complex mathematics problem solving, students struggle—especially at the primary grades (Durnin, Perrone, & Mackay, 1997). One reason students struggle with real-world complex problems is that they have difficulty developing appropriate schemas or problem representations that allow them to identify the necessary solution methods to successfully solve the problems (Chi, Feltovich, & Glaser, 1981; Fuchs et al., 2006; Quilici & Mayer, 1996). Another issue that students face is that they lack the self-regulatory skills necessary to navigate complex problems (Zimmerman & Martinez-Pons, 1990). As previous research has shown, key to successful mathematics problem solving is the self-regulation of one's learning (de Corte, Verschaffel, & Op't Eynde, 2000; Muis, 2004; Schoenfeld, 1994; Zimmerman & Labuhn, 2012). According to Schunk and Ertmer (2000), self-regulated learning is defined as self-generated thoughts, feelings, and actions that are oriented toward learning goals. Central to

self-regulated learning is metacognition (Efklides, 2008; Muis, 2007; Winne & Hadwin, 2008). Students who engage in more metacognitive processes typically achieve better learning outcomes (van der Stel & Veenman, 2010; Zimmerman, 2002).

It is unfortunate that younger learners, especially elementary students, are not very good at self-regulating or monitoring their learning (Butler & Winne, 1995; Zimmerman & Martinez-Pons, 1990). As such, it is imperative to develop learning environments that foster a better understanding of a problem's representation and increase key self-regulatory processes to improve learning outcomes. To address this, in collaboration with two teachers<sup>1</sup> who recently implemented a one-to-one tablet program<sup>2</sup> at their schools, we developed an intervention designed to foster a better problem representation, greater self-regulatory strategy use and better learning outcomes in the context of complex mathematics problem solving. We framed our intervention within the learning by teaching (e.g., Biswas, Jeong, Kinnebrew, Sulcer, & Roscoe, 2010; Biswas, Leelawong, Schwartz, Vye, & The Teachable Agents Group at Vanderbilt, 2005; Palinscar & Brown, 1984; Roscoe, 2014), and self-regulated learning (Greene & Azevedo, 2009; Muis, 2007; Winne & Hadwin, 2008) literatures. Specifi-

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<sup>1</sup> One teacher previously developed her own approach to learning by teaching by having students create concept maps of their understanding of a content area. After students developed their concept maps, students developed teaching videos to explain their understanding of that content area. We used this approach as the foundation for the current study. The teachers also chose the problem as well as the applications for solving the problem.

<sup>2</sup> All students are required to purchase their own iPad for school purposes, which they use on a daily basis for all content areas.

cally, the purpose of our research was to explore the effects of learning by preparing to teach compared with the effects of learning for learning (control condition) on students' problem representations, self-regulatory processes and learning outcomes during complex mathematics problem solving. We describe relevant theoretical and empirical work next.

### Learning by Teaching

Fiorella and Mayer (2013) define learning by teaching as a learning environment in which a student is given the role of the teacher and is asked to teach academic content to others for instructional purposes. Others may include peers or computer-based agents (Biswas et al., 2010; Roscoe & Chi, 2007). Four lines of research have been conducted within the learning by teaching paradigm, including learning by preparing to teach (Annis, 1983; Bargh & Schul, 1980; Fiorella & Mayer, 2013; Renkl, 1995), learning by (actually) teaching (Annis, 1983; Fiorella & Mayer, 2013, 2014), learning through peer tutoring (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001; De Backer, Van Keer, & Valcke, 2012; King, Staffieri, & Adelgais, 1998; Roscoe & Chi, 2007), and teachable agents, such as computer-based agents (Biswas et al., 2005, 2010; Roscoe, Segedy, Sulcer, Jeong, & Biswas, 2013).

We developed an intervention wherein elementary students used tablet applications to solve a complex mathematics problem and created a video wherein they explained how to solve the problem to be used to teach others. As previous research has shown, teaching others can be an effective way to enhance learning (e.g., Biswas et al., 2010; King et al., 1998; Palinscar & Brown, 1984; Peets et al., 2009; Roscoe & Chi, 2007) across a wide range of age groups including college (Annis, 1983), high school (Cloward, 1967; Morgan & Toy, 1970), middle school (Jacobson et al., 2001), and elementary school (Fuchs et al., 1996). That is, through learning by teaching, individuals theoretically learn content more deeply by teaching it to others compared with learning the content just for oneself. However, the reasons for this positive effect on learning remain unclear (Fiorella & Mayer, 2013; Galbraith & Winterbottom, 2011; Peets et al., 2009; Rohrbeck, Ginsburg-Block, Fantuzzo, & Miller, 2003; Roscoe & Chi, 2007), and some research has shown no positive effects (Renkl, 1995) or negative effects (Ehly, Keith, & Bratton, 1987) on learning. In their seminal article, Bargh and Schul (1980) proposed that the expectation of teaching content to others results in a change in the way individuals study that material compared with normal studying for oneself. They argued that to teach, individuals must develop a good understanding of the domain knowledge and then structure that knowledge in a way that can be presented to others. When learning by preparing to teach, students arguably devote more resources toward selecting the most relevant material and organizing it into meaningful representations (Roscoe & Chi, 2007).

Research on learning by preparing to teach provides support for this hypothesis (Annis, 1983; Benware & Deci, 1984; Biswas, Schwartz, & Bransford, 2001; Fiorella & Mayer, 2013). For example, Fiorella and Mayer (2013) explored the relative effects of learning by preparing to teach and by actually teaching on learning. Students were given the task of studying a lesson on the Doppler effect without the expectation of later teaching the material and then took a comprehension test on the same material (control group). Other students were given the same material but

were told they would actually teach the content by preparing a brief video of the material. Half of these students were given the comprehension test immediately after studying (preparation group), whereas the others prepared a lecture and then were given the comprehension test (teaching group). Results revealed that the preparation and teaching groups significantly outperformed the control group on the comprehension test (effect sizes were  $d = .82$  for the difference between the teaching vs. control group, and  $d = .59$  for the difference between the preparation vs. control group). However, in a second experiment with the same design, they found that only the teaching group outperformed the control group on a 1-week delayed test ( $d = .79$ ).

The majority of research that has explored the effects of learning by teaching has been drawn primarily from the peer-tutoring literature. From the peer-tutoring literature, Roscoe and Chi (2007) propose that tutors benefit from instructing others because they must be able to explain the content to others. To explain content well to others requires that tutors are able to evaluate their own understanding, gaps in knowledge, or confusions that arise during learning. They must also be able to recover from those confusions, all of which requires substantial self-monitoring and evaluation (King, 1998). Tutors also need to organize the content in well-structured ways to allow them to provide clear explanations. As such, it is likely during the preparing to teach phase that learning gains occur given that tutors must organize the content in ways that allow them to effectively teach it to others. According to Biswas et al. (2010), this initial structuring of knowledge is likely fostered through self-explanations. Individuals are more likely to engage in these self-explanations to ensure they can teach the content to others. As previous research has shown, when individuals engage in more self-explanation processes, this should facilitate learning (e.g., Chi, DeLeeuw, Chiu, & LaVancher, 1994; Matthews & Rittle-Johnson, 2009) via increased metacognitive processes (Kwon & Jonassen, 2011).

For example, De Backer et al. (2012) explored the role of reciprocal peer tutoring to promote university students' metacognitive knowledge and regulation skills. Students enrolled in an instructional science course engaged in reciprocal peer tutoring over eight sessions with actual course material. Using a pretest-posttest design, students self-reported their metacognitive knowledge and regulation skills. Additionally, a think aloud protocol was used to measure students' actual use of metacognitive strategies (orientation, planning, monitoring, and evaluation of learning). Results at posttest revealed no difference in students' self-reported metacognitive knowledge and strategy use, but differences were observed in actual metacognitive strategies used. Specifically, posttest results revealed significant increases in orientation wherein students were more likely to analyze the task ( $d = 3.12$ ), structure task instructions ( $d = .75$ ), and orient themselves to specific content ( $d = 1.52$ ). Differences in metacognitive monitoring were also observed, wherein students focused more on comprehension monitoring ( $d = 1.72$ ) and progress on task ( $d = 1.67$ ), and on understanding ( $d = .90$ ) and elaborating the text ( $d = 2.29$ ). Similarly, students engaged in more evaluation of their learning outcomes at posttest compared with pretest ( $d = 2.46$ ), but no differences were observed in students' planning activities from pretest to posttest.

In another study, King et al. (1998) assigned seventh graders in pairs to one of three peer-tutoring conditions: explanation only,

inquiry plus explanation, or sequenced inquiry plus explanation. Students engaged in both tutor and tutee roles after teacher-led lessons on systems of the human body. Tests were conducted at pretreatment, posttreatment, and 8 weeks following the posttest to assess students' comprehension of factual material as well as their ability to make inferences and integrate material. Students' metacognitive awareness and self-regulation of their use of the tutoring protocol were assessed using self-report scales, which focused on their understanding of the content, implementation of supportive communication techniques, and how well they explained new material. Results revealed that at posttest and 8 weeks following posttest, there were no differences between groups on factual material learned. However, students who engaged in inquiry plus explanation and sequenced inquiry plus explanation performed better on inference and integration tasks compared with students in the explanation only condition (no effect sizes were reported). It is interesting to note that differences between groups were not found for self-reported metacognition, but there was a significant increase in students' metacognitive awareness and self-regulation improved over time.

Despite results from these and other studies, it is still unclear what the underlying mechanisms might entail with regard to why greater learning gains may occur. For example, in King et al.'s (1998) study, there were no differences in self-reported metacognitive strategies despite theoretical assumptions that increases in metacognitive strategies should occur. For De Backer et al.'s (2012) study, the researchers were not able to make substantive causal claims regarding why there were increases in use of metacognitive strategies given the lack of a control group. One explanation, initially proposed by Roscoe and Chi (2007), is that differences in learning processes and outcomes might arise due to knowledge building versus knowledge telling in tutors' behaviors. Knowledge building is defined as a process of metacognitively monitoring one's own knowledge and understanding, integrating new and prior knowledge, and generating new ideas through inference and reasoning. In contrast, knowledge telling is defined as a process of summarizing or restating source materials with little deep reasoning or reflection. As Roscoe (2014) argued, knowledge telling may occur due to tutors' inadequate evaluation of their own understanding of the material. Indeed, Roscoe found that tutors' comprehension monitoring and domain knowledge, as well as tutees' questions, were significant predictors of knowledge building. However, we argue that one central component that has been overlooked in the learning by teaching literature is the initial structuring of the task and content that occurs during the preparing to teach phase (Bargh & Schul, 1980). To elaborate this, we turn to the self-regulated learning literature.

### Self-Regulated Learning

Students who self-regulate their learning plan how to approach a learning task, set goals, implement strategies to carry out the task, and evaluate progress and products throughout the learning process. Planning, implementation, and evaluation are key phases in Muis's (2007) theoretical framework. Muis's model was chosen given its focus on metacognition as the hub of self-regulated learning, which is central to mathematics problem solving (Jacobse & Harskamp, 2012; Schoenfeld, 1982). Muis's model was also chosen given its focus on the task definition phase of self-regulated

learning, which is influenced by the instructional context provided for a task. We elaborate this below.

### Muis's (2007) Model of Self-Regulated Learning

Based on goal-oriented (Pintrich, 2000) and metacognitively driven (Winne & Hadwin, 2008) models of self-regulated learning, as shown in Figure 1, Muis (2007) proposed a model of self-regulated learning that includes four phases of learning: task definition, planning and goal setting, enactment, and evaluation. According to Muis (2007), in the first phase, task definition, the learner defines the task based on external conditions, such as the instructional context (e.g., instructions provided by teachers to complete the task), task features (e.g., the kind of problem) and internal conditions such as prior knowledge and motivation. It is during this phase of self-regulated learning that learners begin to encode and elaborate the initial givens of a problem and develop a representation of that problem (i.e., develop an understanding of the problem), which are all critical for successful mathematics problem solving (Bédard & Chi, 1992; Chi et al., 1981; Fuchs et al., 2006; Schoenfeld, 1994).

It is important to note that when students are given different instructions to complete a task, their task definitions might differ, which may then lead them to implement different learning strategies to carry out the task (Chevrier, Muis, & Di Leo, 2015; Muis, 2007; Winne & Hadwin, 2008). For example, if students are told they will be given a complex mathematics problem to read first, create a concept map of the problem, solve the problem, and then create a teaching video that explains to others how they solved the problem, their task definitions may differ from students who are told to read the problem, create a concept map, and then just solve the problem. As Bargh and Schul (1980) proposed, students in the teaching condition might define the task as one in which they must develop a good understanding of the mathematics problem and then structure each facet of the problem in a way that can be explained to others. When creating the concept map, students in the teaching condition might include more critical information about the problem, and organize information better in terms of the problem's structure given that the standards these students set for understanding the problem may be higher compared with those set by students in a problem-solving only condition.

These variations in task definitions may then result in differences in the plans and goals that individuals set during the second phase. Planning includes selecting the types of learning strategies to carry out the task and identifying the type of information on which to focus during learning. For example, students in the teaching condition may plan to use more metacognitive strategies to ensure progress on understanding the problem is sufficient and during problem solving to ensure a correct solution to each facet is derived; the specific level of understanding the students set to achieve would be identified as a goal. A goal is modeled as a multifaceted profile of information (Butler & Winne, 1995) and each standard in the profile is used as a basis to compare the products created when carrying out the task.

The third phase begins when a learner implements the learning strategies that were planned to carry out the task. Then, in the last phase, individuals evaluate the successes or failures of each phase or products created for the task, or perceptions about the self or context. Products created during learning are compared with the



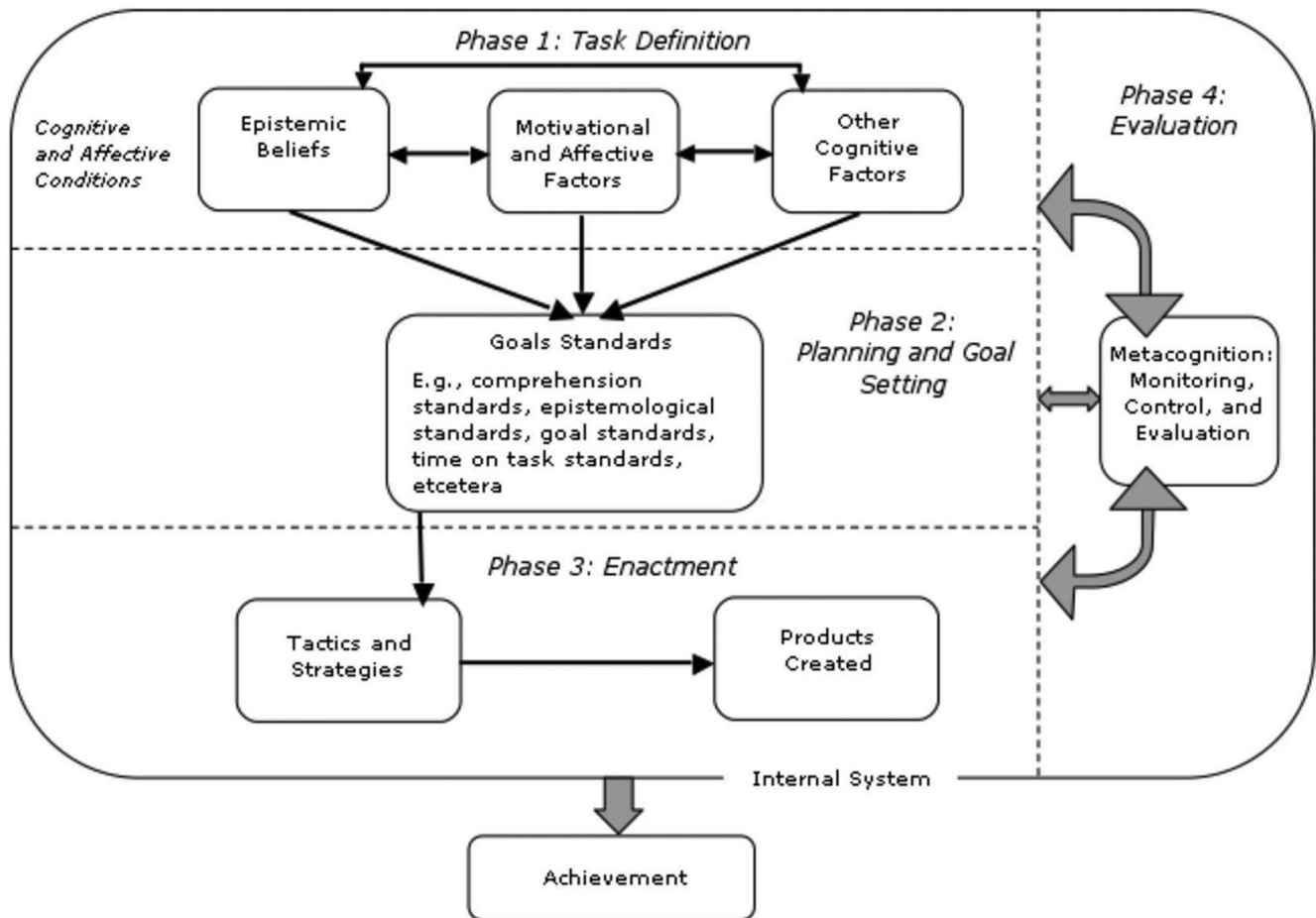


Figure 1. Muis's (2007) model of self-regulated learning. Reprinted from "The Role of Epistemic Beliefs in Self-Regulated Learning, by K. R. Muis, 2007, *Educational Psychologist*, 42, p. 177. Copyright 2007 by Taylor & Francis. Reprinted with permission.

standards set via metacognitive monitoring. If monitoring reveals an inadequacy of one or more of the products created (e.g., an answer is incorrect), a learner may engage in control processes wherein other cognitive strategies are employed to reduce the discrepancy. According to Muis (2007), metacognitive processes can occur during all four phases of self-regulated learning. That is, monitoring, control, and reaction/evaluation can be ongoing throughout the learning process, and goals and plans may also change or be updated as feedback about progress becomes available. Moreover, products created across all four phases can feed into other phases, which reflect the cyclical nature of self-regulated learning in her model.

### The Current Study

Taken together, Muis's (2007) model provides a theoretical explanation with regard to why differences in learning outcomes might occur when learning by teaching versus learning for learning. Within the broader learning by teaching paradigm, research has focused primarily on learning outcomes (e.g., Biswas et al., 2010; Palinscar & Brown, 1984; Peets et al., 2009; Roscoe & Chi, 2007). Few studies have explored self-reported or actual cognitive

or metacognitive processes during learning by teaching (De Backer et al., 2012; King et al., 1998; Roscoe, 2014; Roscoe & Chi, 2007). Moreover, to our knowledge, no study has been conducted to assess what learners' initial understanding of the content entails and how they structure that content in preparation for teaching. Research is needed wherein individuals' task definitions and cognitive and metacognitive processes are traced as they occur during problem solving across each phase of self-regulated learning. As such, the purpose of this research was to explore whether learners' task definitions and self-regulatory processes differed when learning by preparing to teach versus learning for learning in the context of complex mathematics problem solving in a technology-rich learning environment. The research was conducted in a classroom context during regular school time with a sample of elementary students from two different schools.

From a self-regulated learning perspective, learners' task definitions should differ when learning solely for oneself versus when given the task to teach others. To capture individuals' task definitions, concept maps can be used to evaluate what information learners think is important and how they structure that information (Pintrich, Marx, & Boyle, 1993), a method that has previously

been used in research on mathematics (Laturno, 1994; Williams, 1998). That is, students' concept maps can be analyzed for quantity of information as well as how that information is organized. To date, research within the learning by teaching framework has not explored this possibility. As such, our first research question was, Do students' task definitions differ when learning by preparing to teach versus learning for learning? Students were asked to create a concept map that included four features: blue for the title, red for the first step to solve the problem, black for important information, and green for calculations needed to carry out the problem. We hypothesized that students in the learning by preparing to teach condition would include more important information about the problem in their concept map and would hierarchically structure the information better (i.e., green calculations subsumed under related important information or vice versa) than students who were asked to just solve the problem.

Differences in task definitions should then theoretically result in differences in planning and goal setting (Muis, 2007; Winne & Hadwin, 2008). That is, if students define the task as one in which they need to develop a good understanding of the problem to be able to explain to others how to solve it, then these individuals may, for example, plan to use more metacognitive strategies to ensure sufficient progress and understanding compared with students who are told to simply solve the problem. Students in a teaching condition may also set more goals, like ensuring their work is done well, compared with students who just solve the problem.

Based on differences in planning and goal setting, differences should also arise during the enactment and evaluation phases wherein various cognitive and metacognitive strategies are employed (Muis, 2007). As such, our second research question was, Are there differences in the frequency of self-regulatory processes, such as planning and goal setting, cognitive processes, and metacognitive processes, when solving a complex mathematics problem when learning by preparing to teach versus when learning for learning? To capture self-regulatory processes of planning and goal setting, cognitive strategies, and metacognitive strategies, a think-aloud protocol was used (Azevedo, 2005; Greene & Azevedo, 2009; Muis, 2008). We hypothesized that students in the learning by preparing to teach condition would engage in more planning and goal setting, and use more cognitive and metacognitive processes during problem solving compared with students in the control condition (Bargh & Schul, 1980; De Backer et al., 2012).

Our final research question was, Does learning by preparing to teach result in higher levels of mathematics problem solving achievement compared with learning for learning? Given that conceptual understanding of the problem and metacognitive processes are central to successful mathematics problem solving (Chi et al., 1981; Fuchs et al., 2006; Schoenfeld, 1994), we hypothesized that students in the learning by preparing to teach condition would have a higher achievement score on the complex mathematics problem compared with students who just solved the problem.

Finally, given that previous research has found gender differences in self-regulatory processes at the elementary-school level (Zimmerman & Martinez-Pons, 1990), gender was included as a variable. We hypothesized that females would engage in more self-regulatory processes compared with males (Zimmerman & Martinez-Pons, 1990) and, as a result, have a higher achievement score on the mathematics problem (Hyde, Fennema, & Lamon, 1990; Voyer & Voyer, 2014). As previous research has found relations between prior knowledge and self-regulated learning (Zimmerman & Martinez-Pons, 1990), prior knowledge in mathematics was included as a covariate for all analyses.

## Method

### Participants

Eighty-two students were invited to participate from two different schools across four different classrooms, and 78 agreed ( $n = 34$  females, 0% attrition rate, and parents provided consent and students provided assent). Students were from the same English public school board in the province of Quebec, Canada. Both schools were bilingual wherein students spent 50% of their time learning in English and the other 50% learning in French. Seventy-six students were first-language English (EFL), and two were first-language French (FFL) but were fully fluent in English. Of the 78 students, 75 were Caucasian, two were Indo Canadian, and one was African Canadian. See Table 1 for a complete summary of the demographic information.

### Materials

**Demographics.** Demographic information was obtained from the parental consent forms, which included students' age (by date

Table 1  
*Demographics by Frequency or Average*

| School   | Females | Males | EFL | FFL | Age      |           | IEP | LBT | LFL |
|----------|---------|-------|-----|-----|----------|-----------|-----|-----|-----|
|          |         |       |     |     | <i>M</i> | <i>SD</i> |     |     |     |
| School 1 | 22      | 18    | 40  | 0   | 11       | 0.31      | 5   | 20  | 20  |
| School 2 | 14      | 24    | 36  | 2   | 11       | 0.31      | 4   | 19  | 19  |
| Total    | 36      | 42    | 76  | 2   | 11       | 0.31      | 9   | 39  | 39  |

*Note.* Individualized education plans (IEP; adapted or modified) are used to help describe and organize the support measures and personalized follow-up that are necessary to help students with special needs progress in their schooling and to foster their success. The IEP is a collaborative process by which an educational plan is created for a student. This plan involves identifying the needs and strengths of the student, creating short- and long-term goals and objectives, and specifying the accommodations or strategies that will be used to achieve those objectives. For the purposes of this study, coding of students' mathematics solution was the same regardless of IEP status. LBT = learning by teaching; LFL = learning for learning; EFL = English first language; FFL = French first language.

of birth), gender, and primary and secondary languages spoken at home.

**Prior knowledge.** Students' achievement score on a compulsory provincial exam was used to obtain a measure of prior knowledge. The provincial exam was completed 1 week prior to the beginning of the research study (all students in the province must complete this exam, which counts for 30% of their final grade in mathematics). Commencement of the research study was chosen to immediately follow the provincial exam to ensure a valid and standardized assessment of students' prior knowledge. The exam included a series of multiple-choice questions that assessed students' knowledge of the mathematics content covered over the school year. Cronbach's alpha reliability coefficient for the prior knowledge test was .94.

**Complex mathematics problem.** The situational problem *Start Your Engines* was drawn from the 2009 provincial exam. The objective is to have students develop a coherent solution to a situational problem that meets the following conditions: (a) the procedure required to solve the situational problem is not obvious, because it involves choosing a significant number of previously acquired mathematical concepts and processes and using them in a new way; (b) the situation focuses on obstacles to overcome, which requires various learning strategies; and, (c) the instructions do not suggest a procedure to be followed or the mathematical concepts and processes to be used (Ministère de l'éducation, du loisir et du sport, 2009). In the first phase of problem solving, students read the problem and then developed a concept map of the problem (see next section). In the second phase, students were required to solve the problem, and show all steps and decisions made along the way. For this particular problem, students had to: create a seven-sided polygon for the racetrack design that ranged in length between 4.5 km to 5 km; include at least one acute angle, one obtuse angle, and one angle greater than 180°; create spectator areas with 15 squares per section to seat 120,000 spectators; draw a starting line frieze pattern that was one third white, reflected twice; and calculate the cost of the paint for the starting line given indications of the price per unit.

**Mathematics achievement.** A rubric (see Appendix A) was developed to score each student's solution to the complex mathematics problem, and total score on each student's solution was used as the measure of mathematics achievement. This particular problem was graded based on the inclusion of all required elements of the task as listed above. Students' calculations for each facet of the problem were also graded. Each element of the problem was given a particular value, and full points were awarded for successfully completing each element. Partial points were given when aspects were missing, and no points were given if an element was completely missing or wrong. For example, if a student created a six-sided polygon, he or she was awarded six points, rather than the full seven points (i.e., one point for each side, but one point was taken off for each additional side over seven). For the perimeter, students were given four points if the track measured between 4.5 and 5.0 km. Two points were awarded if the track was within 0.5 above or below the range. Calculations were given full points if done correctly, and partial points were given if a minor mistake was made (e.g., one number was copied incorrectly from the racetrack to the calculations page). The total number of points to be earned was 50. Krista R. Muis and Cynthia Psaradellis coded 10 students' solutions (randomly selected) to

establish interrater agreement. Interrater agreement was assessed for each facet of the problem (e.g., whether both raters awarded 7 points for the seven-sided polygon, four points for the perimeter). Interrater agreement was 100% ( $\kappa = 1.0$ ). Cynthia Psaradellis then coded all remaining solutions. Coding was blinded to ensure no bias.

**Concept map.** To assess students' understanding of the problem, that is, their task definitions, students used the tablet application *Popplet* to create a concept map (students had been using this concept mapping tool for various content areas for seven months at one school, and for one month at the other school). Students were provided specific criteria developed by the teachers to create their concept map for the mathematics problem. Criteria included using four different colored borders to represent various facets of the problem. Students were told to use a black border to represent important information, a green border for calculations needed to solve various aspects of the problem, a red border for the first question that needed to be answered (some aspects of the problem needed to be solved first before others, so order was important in some instances), and a blue border for the title of the problem. For important information, students were asked to create one popple (i.e., concept bubble) for each piece of important information, which they included inside the popple. For calculations, students were similarly asked to create a popple for each calculation they needed to carry out, and were asked to insert the description of the calculations but not actually solve the problem within the popple. Moreover, prior to the study, the teachers trained the students to create the popplets using the four colors and to organize the information whereby calculations and associated important information are linked.

**Coding the concept map.** To score the students' concept maps, a rubric was developed (see Appendix B) that included all aspects of the problem. An expert concept map was also created to allow for ease of comparison (see Figure 2). As shown in Figure 2, there were four main aspects of the problem: racetrack design, spectator area, starting line frieze pattern, and cost of starting line paint. We allowed for flexibility in the hierarchical structure of the concept map wherein students could organize red popples first followed by black or green, or vice versa. Key to coding the organizational structure was the information that was linked as well as each popple's color. For example, there were six possible questions students could solve first (coded in red), and students were given one point for correctly identifying one of the six as the first question. If, however, students indicated two first steps (two red popples), they lost 0.5 marks. For each of the four aspects of the problem, there were a specific number of popples that students could create related to that aspect of the problem. Each of those popples was further color-coded as green or black. Students were given one point for each correctly color-coded popple (i.e., both content and color had to match). If a popple was incorrectly colored (e.g., correct calculation description but the popple was black instead of green), students lost 0.5 of a mark. For organization of the concepts, for each correctly linked popple students were given 0.5 of a mark (marks were not deducted for incorrect links). Total possible score on the concept map was 20. Krista R. Muis and Cynthia Psaradellis then coded 10 concept maps to establish interrater agreement based on scores for each aspect of the problem. Interrater agreement was 100% ( $\kappa = 1.0$ ). Given high inter-



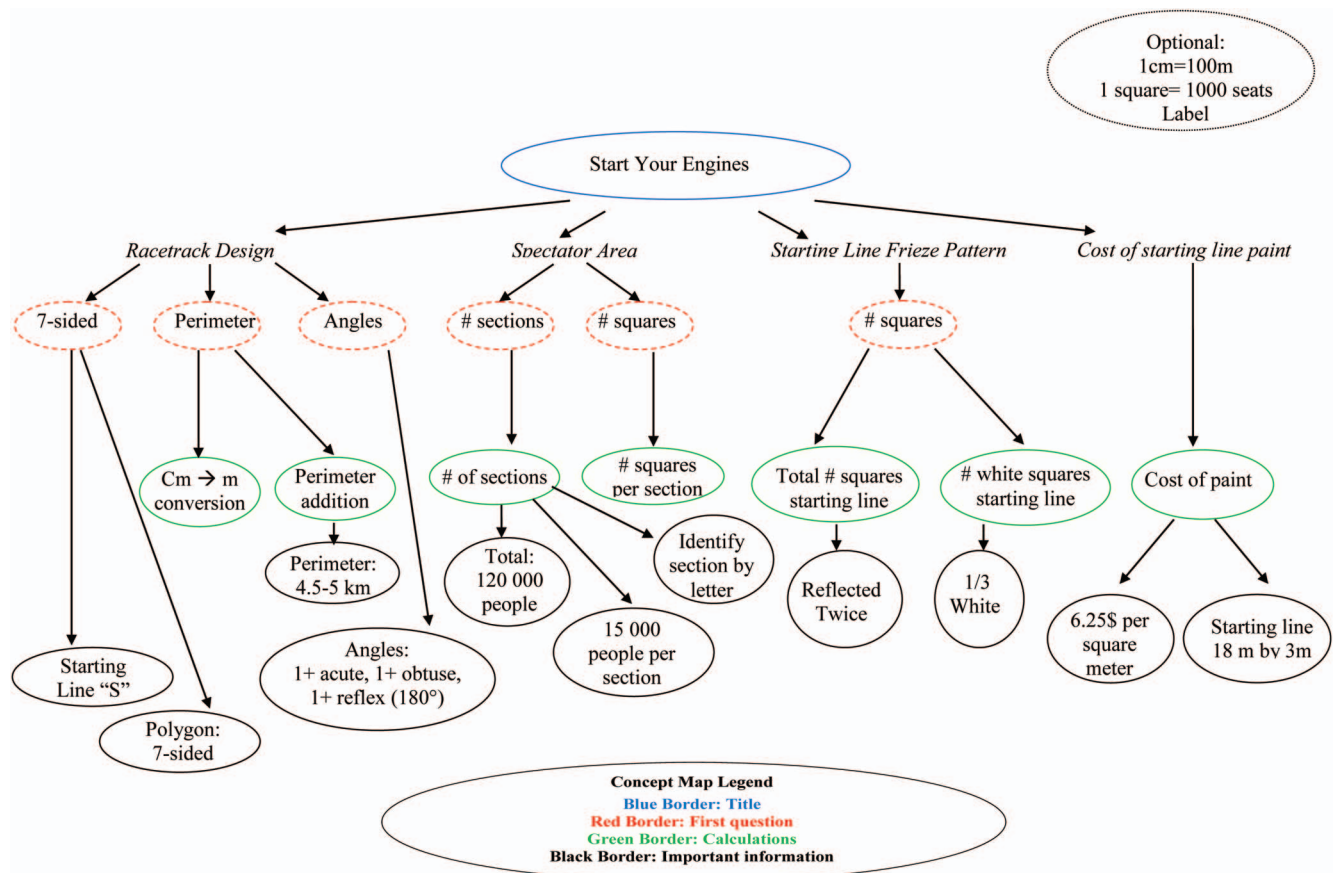


Figure 2. Expert concept map. See the online article for the color version of this figure.

rater agreement, Cynthia Psaradellis then coded the remaining concept maps. Coding was blinded to ensure no bias.

**Self-regulatory processes.** A think-aloud protocol (Type I protocol; see Ericsson and Simon, 1998) was used to capture students' self-regulatory processes as they read the problem, developed their concept map, and solved the complex mathematics problem. Students were instructed to state out loud whatever came to mind. According to Ericsson and Simon (1998), a concurrent Type I think-aloud protocol, which involves thinking out loud while completing a task, does not change the sequence of thoughts and does not affect performance. As such, think-aloud protocols provide a more accurate assessment of individuals' self-regulatory processes as they occur, compared with retrospective self-reports of strategies used during problem solving (see Winne, Jamieson-Noel, & Muis, 2002).

**Coding of self-regulatory processes.** Students' think-aloud protocols were transcribed verbatim. Think-aloud protocols ranged from 90 min to 4.5 hr, which resulted in 1,288 single-spaced pages of text. Greene and Azevedo's (2009) self-regulatory processes coding scheme, as well as Schoenfeld's (1982) and Muis's (2007) theoretical models were used as guides to develop a micro-macro-level coding scheme specific to mathematics problem solving. Macrolevel codes are general self-regulatory processes whereas microlevel codes are specific self-regulatory processes. Six macrolevel processes considered from Schoenfeld's (1982) model in-

cluded reading, analyzing, exploring, planning, implement, and verifying. Greene and Azevedo's (2009) model included five macrolevel and 35 microlevel processes. The macrolevel processes included planning, monitoring, strategy use, as well as handling of task difficulty and demands, and interest. From Muis's (2007) model, we considered four macrolevel processes, which reflected the four phases of her model: task definition, planning and goal setting, enactment, and evaluation.

Based on the codes developed from these three models, five trained research assistants and Krista R. Muis then coded two transcripts (102 pages) to further refine the coding scheme and establish interrater agreement. Interrater agreement was established at 82%, and disagreements were resolved through discussion. Four more weeks were spent refining the coding scheme, and 20 microlevel codes emerged and were categorized along four macrolevel codes based on Muis's (2007) model: task definition, planning and goal setting, enactment, and monitoring and evaluation. Definitions of each of the macrolevel and microlevel codes, along with examples drawn from the transcripts are presented in Table 2.

Once these codes were established, Krista R. Muis then selected three of the longest transcripts from each of the four classes, plus one of the shorter transcripts from each class to ensure comparability across length. The two original protocols used to develop the coding scheme were included in the 16 transcripts chosen, which

Table 2  
*Self-Regulated Learning Coding Scheme for Mathematics Problem Solving*

| Level (macro)/micro                              | Definition  | Examples   |
|--|---|--|
| Level 1: Task definition                         | A learner generates a perception about the task, context, and the self in relation to the task. External and internal conditions play a major role.   | Prior knowledge activation, beliefs, motivation, and knowledge of strategies are activated during this level.  |
| Prior knowledge activation                       | Searching for or explicitly recalling relevant prior knowledge.   | "A reflex angle. That's more than 180 degrees."<br>"5 km, which is short for kilometer."   |
| Identifying important information                | Recognizing the usefulness of information.  | "So that will help us figure out how many people are in each row . . ."  |
| Level 2: Planning and goal setting               | The learner begins to devise a plan to solve the problem and sets goals.  | e.g., planning to use means-ends analysis, trying trial and error, identifying which part of the problem to solve first, solving it within a specific amount of time.  |
| Making/restating a plan                          | Stating what approach will be taken, what strategy will be used to solve the problem, or what part of the problem will be solved in some sequence. This includes restating plans.                             | "First, I have to figure out how many are in each row, then I can figure out how many people fit in each row to fit 120,000 people."<br>"Let's just do trial and error."   |
| Setting/restating a goal                         | A goal is modeled as a multifaceted profile of information, and each standard in the profile is used as a basis to compare the products created when engaged in the activity. This includes restating goals.  | "We have to have an acute angle, obtuse angle, and one reflex angle."<br>"I don't want to spend too much time figuring out the track." [time goal]<br>"I want to make sure my calculations are neat."  |
| Level 3: Enactment                               | Enactment occurs when the learner begins to work on the task by applying tactics or strategies chosen for the task.   |  |
| Hypothesizing                                    | Making predictions.   | "The next one is probably going to tell us the information about the design."  |
| Summarizing                                      | Summarizing what was just read in the problem statement.  | "Next, the spectator seating area, must be divided into sections each section must have seats for 15,000 people. So there, each section has 15,000 people."<br>"The starting line must be painted with a frieze pattern, this pattern is a rectangular design that has to be, that has been reflected twice, so it has to be reflected twice."   |
| Help seeking                                     | Asking for help from a teacher, peer, or other source.<br>Help seeking for information VERSUS help seeking for evaluation.  | [turns to teacher and asks a question] "But what if my track isn't exactly 5 km?"<br>"So we're supposed to do something like this?"<br>"Is this correct?"  |
| Coordinating informational sources               | Using other sources of information to help solve the problem.   | "Let's go back to our popplet." [Popplet includes the concept map, and learner is going back to the concept map he created to help solve the problem].   |
| Level 3: Enactment continued                     | Enactment occurs when the learner begins to work on the task by applying tactics or strategies chosen for the task.   |  |
| Highlighting/labeling /coloring/ drawing/writing | Highlighting information, labeling information as part of the problem-solving process, or taking notes in reference to the problem.<br>Making a drawing to assist learning or as part of solving the problem. | "We can put the starting line just like right there." [labeling]<br>[you can hear the learner's pencil] "So its two sides, 2 sides, 3, kind of look like a good drawing [evaluating quality of drawing], 4,"<br>"This is a reflex angle."<br>[adding up the sides] "10 so that's like 1 km plus 1 km and 400 meters . . ."<br>"4.4 plus 3.1 plus . . . equals . . ."<br>"I'm just going to re-read this . . ." |
| Calculating/measuring                            | Solving equations, measuring, or other similar features.  |  |
| Re-reading                                       | Re-reading a section of the problem, word for word. Important that it is word for word, otherwise it is summarizing.  |  |
| Making inferences                                | Making inferences based on information read or products created from solving the problem.<br>(self-explanation)<br>Explaining why something was done. Key word is <i>because</i> .                            | "So it doesn't say it has to be irregular or regular."<br><br>"I'm just, I'm multiplying 18 by 6.25 [calculating] because there are 6.25 per white squares." [self-explanation]  |

(table continues)

Table 2 (continued)

| Level (macro)/micro  | Definition  | Examples   |
|--|---|--|
| Goal-directed search<br>Level 4: Monitoring and evaluation | Various types of reactions and reflections are carried out to evaluate the successes or failures of each level or products created for the task, or perceptions about the self or context. Reaction and reflection also includes judgments and evaluations of performance on a task as well as the attributions for success or failure. | "I'm looking for another thing that might be useful."<br>Products created are compared to the standards set via metacognitive monitoring. Monitoring and evaluation can include any facet listed above (e.g., progress, motivation, plans, goals, strategies, products like answers or drawings made). |
| Self-questioning   | Posing a question.  | "But how much is that?"<br>"What is the most important thing?"<br>"So how do we turn meters into km?"  |
| Monitoring   | Monitoring something relative to goals.   | "I'm not sure there is a reflex angle in my drawing. Let me check."<br>"I might forget that 'each section must have seats for 15,000 people.'"<br>[learner is counting the number of sides for the polygon]<br>"So we have 1 side, 2 side, 3 side, 4 side, 5 side, 6 side, 7 sides."                   |
| Judgment of learning                                       | Learner is aware that something is unknown, not fully understood, or difficult to do.   | "That would be an acute angle, which is kind of hard to draw, this is hard to draw."<br>"I don't really understand this."<br>"I'm not sure."<br>"This is going to be very hard to figure out."   |
| Level 4: Monitoring and evaluation continued               | Various types of reactions and reflections are carried out to evaluate the successes or failures of each level or products created for the task, or perceptions about the self or context. Reaction and reflection also includes judgments and evaluations of performance on a task as well as the attributions for success or failure. | Products created are compared with the standards set via metacognitive monitoring. Monitoring and evaluation can include any facet listed above (e.g., progress, motivation, plans, goals, strategies, products like answers or drawings made).  |
| Self-correcting  | Correcting one's mistakes.  | "Here are 4 km. Not 4 km. Sorry, 400 meters."<br>"So the first thing was the track had to be has to be a 4-sided [summarizing], not a 4-sided sorry a 7-sided polygon [self-correcting]."<br>"Oops, that was actually an obtuse angle."  |
| Evaluation   | Judging whether goals have been met, whether a particular strategy is working, whether the answer is correct, whether the work is neat, etc. Judgment of all facets that fall under monitoring.   | After counting the number of sides of the polygon, the learner states, "Yes, I have 7 sides. Okay, we're good."<br>"I measured the wrong thing by accident."<br>"That's not very neat."  |
| Control  | Changing strategy when monitoring or evaluation results in a determination that goal has not been met.  | [after judging that polygon was not 7-sided] "I'm just going to erase this. It has to be a 7-sided polygon so lets do a different one."  |
| Task difficulty  | Statements reflecting the difficulty or easiness of a task.   | "This is difficult."<br>"This is easy."  |

resulted in a total of 472 pages of transcripts. Krista R. Muis and five research assistants then spent an additional 8 weeks working together to establish and modify the coding scheme, coding, and then recoding the transcripts until an acceptable level of interrater reliability was achieved. At the end of this process, interrater agreement was 85% for the 16 transcripts ( $\kappa = .68$  overall, with similar values for each of the macro categories, although planning and goal setting was the highest at .75).

The frequency with which the original four macrolevel and 20 microlevel codes occurred was then calculated. Following Greene and Azevedo (2009), codes with low frequencies were removed (e.g., averages less than three over a 4-hr period). The modified coding scheme consisted of three macrolevel and 12 microlevel codes. The first macrolevel code was labeled planning and goal setting in which two microlevel codes, plans and goals, were

included. The second macrolevel code was cognitive processes, which included summarizing, help seeking for information, help seeking for evaluation, calculating, coloring, and rereading. The third macrolevel code was metacognition, which included monitoring, judgment of learning, evaluation, and control. The frequency with which students used each of the microlevel codes were then summed within each of the three macrolevel codes, which were used in subsequent analyses. Coding was blinded to ensure no bias.

## Procedure

Students at each school were randomly assigned to one of the two conditions: learning by preparing to teach or the control condition (learning for learning). One day prior to solving the

mathematics problem, Krista R. Muis trained students to think out loud (a script was used to ensure identical training for all students). Thinking out loud was described as, “*saying out loud everything that you say to yourself silently*.” The students then heard a practice think-aloud audio file that modeled what not to do followed by an appropriate think out loud example. That is, the second attempt included intermediate steps and spontaneous thoughts. Students then spent 15 min practicing thinking out loud with a short problem.

The next day, students were given the problem to solve. To ensure that all students were thinking out loud, five trained research assistants and Krista R. Muis were present in the classroom to prompt students to continue to think out loud if they were silent for more than 5 seconds (a ratio of approximately four supervised students per prompter, with students sitting at round tables). Students were told that the problem was to be treated as if it were an exam, and they were not allowed to work together or copy each other’s work during problem solving. Barriers were used to minimize noise levels and cheating (which is normal practice for tests done in class). Earpods with microphones were used to capture students’ thought processes, which were recorded on the tablets using the application *Evernote*. Microphones were placed close to students’ mouths to ensure high quality recordings.

In the learning by preparing to teach condition, students were instructed to first read the problem, create a concept map of the problem, solve the problem, and then create a video, using *Doodle Cast*<sup>3</sup> (an application designed for teaching a lesson), to teach other students how to solve the problem. Students in this condition were told that when they developed their teaching video they needed to explain all steps involved, explain how they solved each step, and were told they could use all materials they created to solve the problem (e.g., their concept maps, calculations). In the control condition, students were instructed to first read the problem, create a concept map of the problem, and then solve the problem. Students in both conditions were told on many occasions not to tell other students about their task and were seated in different classrooms to ensure no confounding. Both conditions were conducted each day consecutively and counterbalanced across times such as late morning or early afternoon.

Once students were told the explicit instructions for their condition, audio recording began, and students read the problem out loud. In both conditions, students then completed the concept map using *Popplet*, a concept-mapping application, to construct their concept maps, and then solved the problem. All work was done during regular class time, and time spent on task for both learning conditions was equivalent (approximately 1.0 to 1.5 hours each day over 3 to 4 days). Students took as much time as they needed to solve the problem but to ensure equivalent time on task across the two conditions, time on task was measured for each student (as a function of the length of their think-aloud recordings). During problem solving, students recorded their calculations and notes in *Noteshelf*, and were provided several copies of the racetrack design on which to draw their work. Once students solved the problem, they were asked to submit all work, which was then scored for correctness. That is, all students’ work was scored and coded after completion of the learning phase. No materials were collected after this phase (i.e., videos were not analyzed for correctness). To thank students for their participation, each student received a \$15 iTunes card.

Table 3  
Zero-Order Correlations

| Variables                    | 1     | 2     | 3     | 4     | 5     |
|------------------------------|-------|-------|-------|-------|-------|
| 1. Prior knowledge           |       |       |       |       |       |
| 2. Planning and goal setting | .13   |       |       |       |       |
| 3. Cognitive strategies      | .40** | .39** |       |       |       |
| 4. Metacognitive strategies  | .26** | .48** | .58** |       |       |
| 5. Concept map               | .29*  | .14   | .37** | .40** |       |
| 6. Mathematics achievement   | .46** | .21   | .45** | .46** | .33** |

\*  $p < .05$ . \*\*  $p < .01$ .

After submission of their work, students in the learning by preparing to teach condition developed their teaching video with *Doodle Cast* using the same materials they developed for solving the problem. The researchers did not collect the videos given that the focus was on learning by preparing to teach; however, the teachers used them for pedagogical purposes.

## Results

### Preliminary Analyses

Skewness and kurtosis values were examined for normality for all variables. For kurtosis, all variables were within an acceptable range (using Tabachnick & Fidell’s, 2013, criteria of  $< |3|$ ). For skewness, with the exception of planning and goal setting (5.29), variables were within an acceptable range. Because the measurement of plans and goals was on a ratio scale, scores were not transformed (see Tabachnick & Fidell, 2013).

We then examined whether there were differences across schools on each of the variables, and ICCs were calculated. No differences were found on any of the variables (all  $p > .05$ ), and all ICCs were below .05. As such, the two schools were combined into one overall sample. Group differences as a function of learning condition were then examined for time on task and prior knowledge. As expected, there were no differences between the learning by preparing to teach intervention group compared with the control group on time on task (average time on task for both groups was slightly less than 2 hr,  $p = .78$ ) or on prior knowledge ( $p = 1.00$ ). However, as expected, gender differences were found for prior knowledge,  $F(1, 76) = 4.61$ ,  $p < .05$ ,  $\eta^2 = .06$ , whereby females had a higher level of prior knowledge than males. As such, prior knowledge was used as a covariate in all subsequent analyses.

### Correlations Among Variables

Table 3 presents the zero-order correlations for all variables: prior knowledge, planning and goal setting, cognitive strategies, metacognitive strategies, concept map score, and mathematics problem solving achievement score. Prior knowledge was significantly positively related to concept map score,  $r(69) = .29$ ,  $p < .05$ ; cognitive strategies,  $r(78) = .40$ ,  $p < .001$ ; metacognitive

<sup>3</sup> All 78 students had been trained to use *Doodle Cast* 3 weeks prior to the study wherein they developed a teaching video using different content (i.e., social studies). This was done with teachers and was not associated with this study in particular.



Table 4  
Means and Standard Deviations for Each Variable as a Function of Learning Condition by Gender (Raw Frequencies Shown Unless Otherwise Noted)

| Variables                      | Learning by preparing to teach |           | Learning for learning |           |
|--------------------------------|--------------------------------|-----------|-----------------------|-----------|
|                                | <i>M</i>                       | <i>SD</i> | <i>M</i>              | <i>SD</i> |
| Planning and goal setting      |                                |           |                       |           |
| Females                        | 74.59                          | 47.68     | 80.38                 | 55.06     |
| Males                          | 30.41                          | 22.40     | 29.77                 | 25.02     |
| Total                          | 49.67                          | 41.56     | 51.08                 | 47.18     |
| Cognitive strategies           |                                |           |                       |           |
| Females                        | 114.65                         | 41.99     | 115.50                | 56.21     |
| Males                          | 76.32                          | 56.31     | 61.00                 | 31.27     |
| Total                          | 93.03                          | 53.53     | 83.95                 | 50.79     |
| Metacognitive strategies       |                                |           |                       |           |
| Females                        | 81.94                          | 35.64     | 60.25                 | 26.04     |
| Males                          | 51.50                          | 28.13     | 42.14                 | 23.46     |
| Total                          | 64.77                          | 34.73     | 49.76                 | 25.88     |
| Concept map (in %)             |                                |           |                       |           |
| Females                        | 49.13                          | 21.76     | 43.50                 | 16.09     |
| Males                          | 40.55                          | 14.53     | 30.00                 | 14.22     |
| Total                          | 44.23                          | 18.21     | 36.14                 | 16.35     |
| Mathematics achievement (in %) |                                |           |                       |           |
| Females                        | 91.72                          | 6.22      | 86.41                 | 11.51     |
| Males                          | 85.89                          | 10.96     | 76.55                 | 11.76     |
| Total                          | 88.43                          | 9.55      | 80.70                 | 12.51     |

strategies,  $r(78) = .26, p < .01$ ; and mathematics achievement,  $r(78) = .46, p < .001$ . Cognitive strategies were significantly positively related to concept map score,  $r(67) = 0.37, p < .01$ , as well as with mathematics achievement,  $r(76) = 0.45, p < .01$ . Metacognitive strategies were significantly positively related to concept map score,  $r(67) = 0.40, p < .01$ , and mathematics achievement,  $r(76) = 0.46, p < .01$ . Concept map score was significantly positively related to mathematics achievement,  $r(69) = 0.33, p < .01$ . These results suggest that students who used more cognitive and metacognitive strategies developed better concept maps and achieved higher score in mathematics problem solving. Moreover, students who developed better concept maps had higher levels of achievement in mathematics problem solving.

### Main Analyses Overview

To answer the first research question, whether students' task definitions differ when learning by preparing to teach versus learning for learning, a two-way analysis of covariance (ANCOVA) was used. The two independent variables included the learning condition (learning by preparing to teach vs. control condition) and gender (male vs. female). The dependent variable was students' score on the concept map, which was scored for both content and organization of concepts. For the second research question, whether there are differences in the frequency of self-regulatory processes when learning by preparing to teach versus when learning for learning, a two-way multivariate analysis of covariance was used. The two independent variables were learning condition (learning by preparing to teach vs. control condition) and gender (male vs. female). The dependent variables were planning and goal setting, cognitive learning strategies, and metacognitive strategies. For the third research question, whether students in the learning by preparing to teach condition attained higher levels of mathematics problem

solving achievement compared with students in the control condition, a two-way ANCOVA was used. The two independent variables were learning condition (learning by preparing to teach vs. control condition) and gender (male vs. female). The dependent variable was students' total score on the mathematics problem. For all analyses, the covariate was students' prior knowledge and, with the exception of the analysis for differences in planning and goal setting (power was only .47), all analyses achieved a power of .72 or higher.

### Students' Understanding (Task Definitions) of the Problem

Table 4 presents the means and standard deviations for both conditions as a function of gender for all variables. The first research question addressed whether there were differences in students' understanding of the problem as a function of learning condition, controlling for prior knowledge. To assess this, students developed a concept map of the problem that was coded for both content and organization of that content. Examples of students' concept maps are presented in Figures 3 and 4. As shown in Figure 3, this particular student included the title and all important information, but failed to include the first step needed to solve the problem (red popple) and several popples were colored black when they should be colored green. Despite the missing aspects of the concept map, the organizational structure of the problem was accurate. In Figure 4, this particular student included five of the seven calculation popples (in green), the first step (red), and all important information. However, the concept map was not perfectly structured.

Analyses of students' concept map scores revealed a main effect of learning condition,  $F(1, 63) = 4.30, p = .042, \eta^2 = .064$ , a main effect of gender,  $F(1, 63) = 6.03, p = .017, \eta^2 = .087$ , but



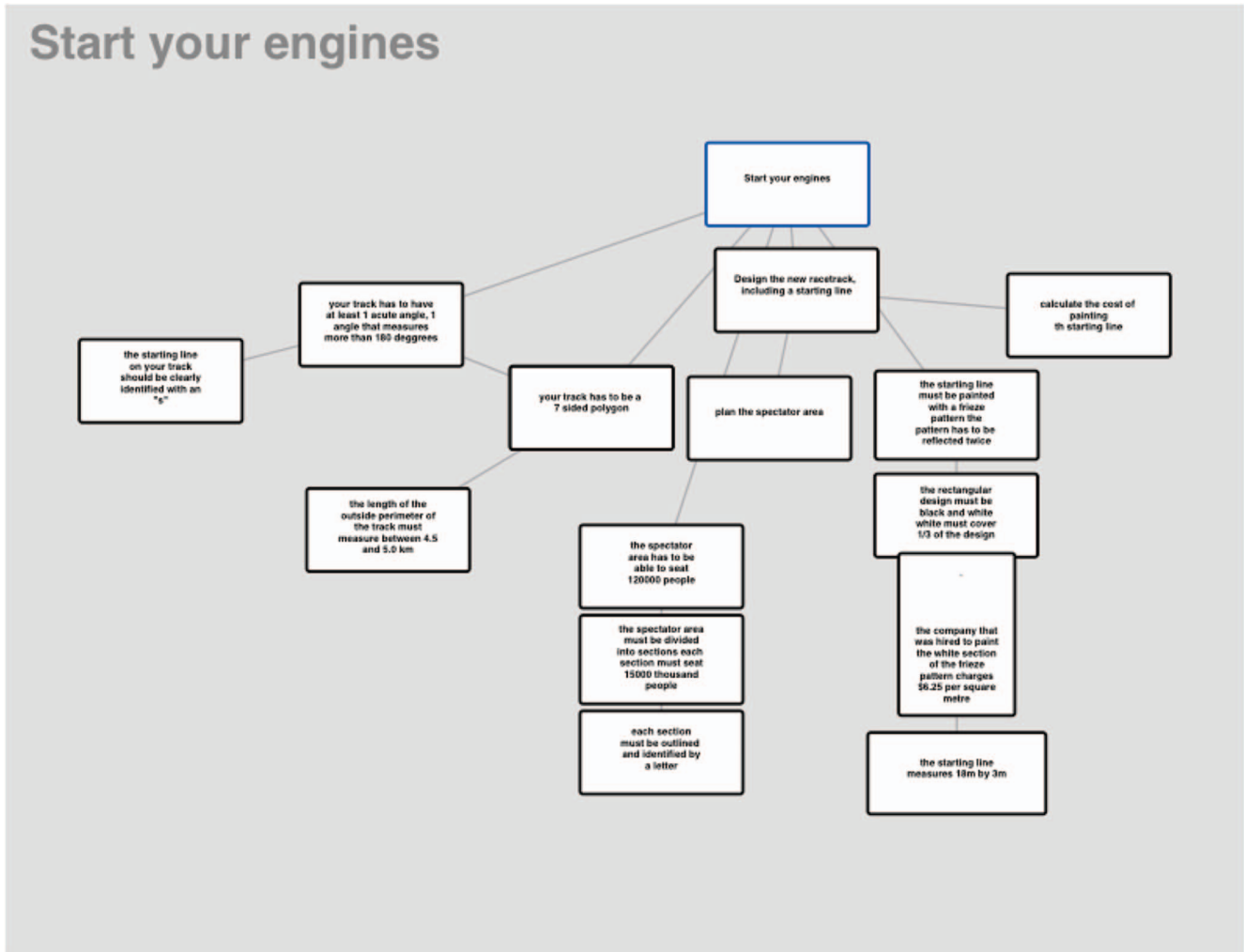


Figure 3. Sample student concept map. See the online article for the color version of this figure.

no condition by gender interaction  $F(1, 63) = .30, p > .05$ . Specifically, consistent with our hypotheses, students in the learning by preparing to teach condition developed a more detailed and better conceptually organized concept map compared with students in the control condition, and females developed a better concept map than males.

### Self-Regulatory Processes

The second research question addressed whether there are differences in the frequency of self-regulatory processes when solving a complex mathematics problem as a function of learning condition. The omnibus multivariate test was significant,  $F(3, 72) = 54.19, p < .001$ , Wilk's  $\Lambda = .30, \eta^2 = .70$ . Analyses of students' planning and goal setting revealed a main effect for gender,  $F(1, 72) = 26.53, p < .001, \eta^2 = .27$ , but no effect for learning condition,  $F(1, 72) = .12, p > .05$ , and no condition by gender interaction,  $F(1, 72) = .060, p > .05$ . Specifically, females had higher frequencies of planning and goal setting than males.

Analyses of students' cognitive learning strategies revealed a main effect of gender,  $F(1, 72) = 16.03, p < .001, \eta^2 = .18$ , but

no effect of condition,  $F(1, 72) = .43, p > .05$ , and no condition by gender interaction,  $F(1, 72) = .47, p > .05$ .<sup>4</sup> Specifically, females had higher frequencies of cognitive learning strategies than males. Finally, analyses of students' metacognitive strategies revealed a main effect of learning condition,  $F(1, 72) = 5.93, p = .017, \eta^2 = .076$ , a main effect of gender,  $F(1, 72) = 11.46, p < .001, \eta^2 = .14$ , but no condition by gender interaction,  $F(1, 72) = 1.16, p > .05$ . Specifically, as hypothesized, students in the learning by preparing to teach condition engaged in more metacognitive processing strategies compared with students in the control condition, and females had higher frequencies of metacognitive strategies than males.

<sup>4</sup> We also assessed differences across groups for each cognitive strategy to evaluate whether students in the intervention condition used different kinds of strategies compared with students in the control condition. No differences were found.

## Start your Engines

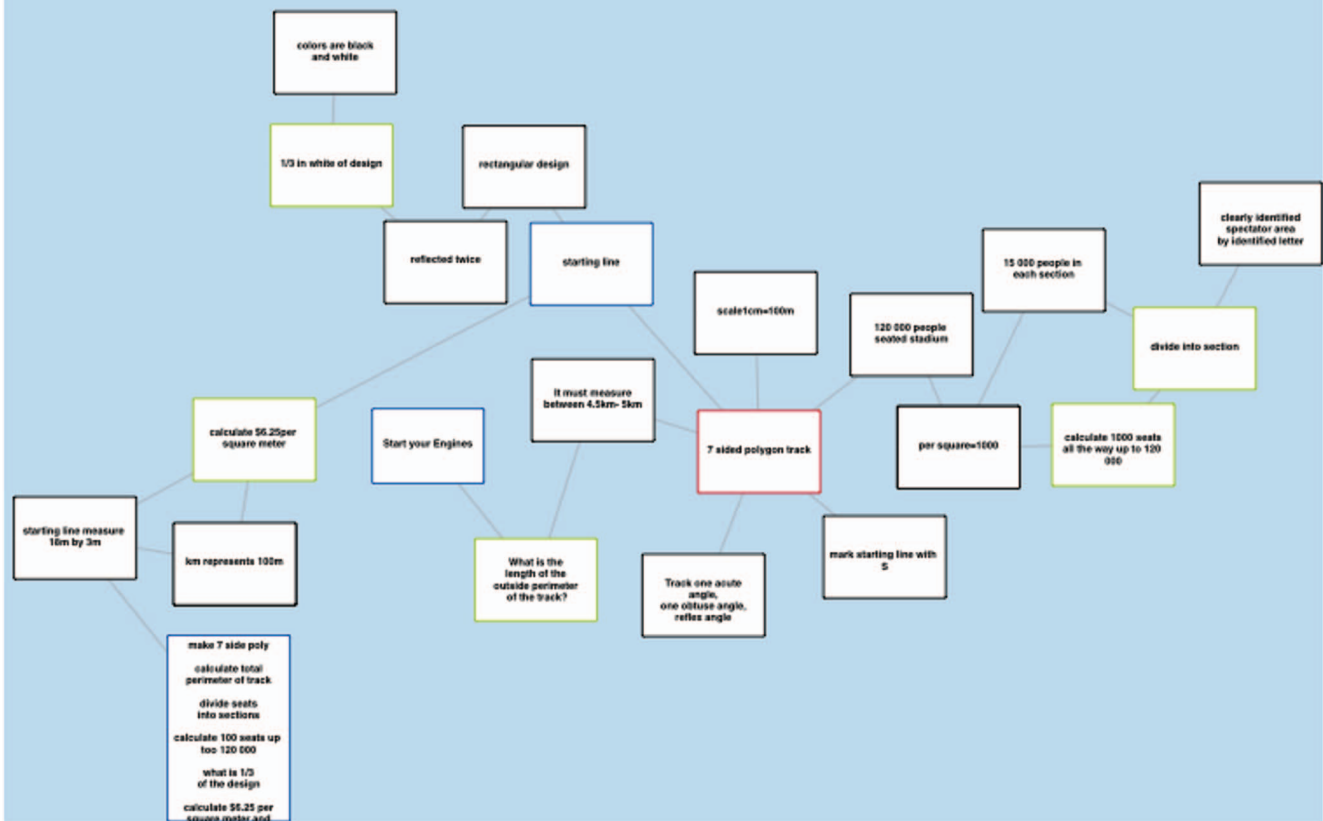


Figure 4. Sample student concept map. See the online article for the color version of this figure.

### Mathematics Problem-Solving Achievement

The third question addressed whether students in the learning by preparing to teach condition obtained higher levels of mathematics problem-solving achievement compared with students in the control condition. Analyses of students' scores revealed a main effect of learning condition,  $F(1, 72) = 9.37, p = .003, \eta^2 = .12$ , a main effect of gender,  $F(1, 72) = 8.73, p = .004, \eta^2 = .11$ , but no condition by gender interaction,  $F(1, 72) = .60, p > .05$ . Specifically, as hypothesized, students in the learning by preparing to teach group had higher levels of achievement compared with students in the control group, and females had higher levels of achievement than males. We discuss the theoretical and practical implications of these results next.

### Discussion

We developed an intervention grounded in the learning by teaching paradigm in the context of complex mathematics problem solving within a technology-rich learning environment. Our goals were twofold. First, we assessed whether the intervention would foster a greater understanding of the problem and increase self-regulatory processes and learning outcomes compared with a control condition. Second, we empirically evaluated the specific

mechanisms theoretically responsible for improvement in learning during the first phase of learning by teaching, that is, learning by preparing to teach. We hypothesized that students in the learning by preparing to teach intervention group would develop a better conceptualization of the problem via the concept maps they created during the initial phase of problem solving compared with students in the control group. We also hypothesized that students in the learning by preparing to teach group would engage in more planning and goal setting, and use more cognitive and metacognitive strategies during problem solving compared with students in the control group.

Results revealed that the intervention designed within the learning by teaching paradigm effectively increased students' understanding of the problem, metacognitive processes, and learning outcomes during complex mathematics problem solving, but no differences were found in students' planning and goal setting, or in the frequency or type of strategies used to solve the problem. In addition, females developed a more complex concept map, had higher frequencies of planning and goal setting, cognitive learning strategies, and metacognitive strategies, as well as higher levels of achievement compared with males. We discuss each of these results in the context of theoretical and educational implications next.

## Theoretical Implications

**Understanding of the problem.** To date, the majority of previous research within the learning by teaching paradigm has focused on the effects of learning by teaching on learning outcomes (Biswas et al., 2010; Palinscar & Brown, 1984; Peets et al., 2009; Roscoe & Chi, 2007). Although some studies have explored whether learners organize and structure the content differently when learning by teaching, interpretations of these differences were based on learners' retrospective self-reports, on transfer tasks that required a better organizational structure of the content for greater performance, or on delayed performance tests (see Fiorella & Mayer, 2013). To better understand why learning by preparing to teach can benefit learners, a direct test of learners' organizational structure of the content was necessary. Our research addressed this gap in the literature. Importantly, results from this research provide evidence that, when asked to develop a concept map of the problem, learners do indeed organize content better when learning by preparing to teach versus when solely learning for learning.

That is, in our study, students in the teaching condition included substantially more information about the problem (given the large effect size; see Lipsey et al., 2012) and linked concepts better than students in the control condition. Specifically, the mean for the intervention group was higher than 69.15% of the scores in the control group given the overall 8% difference between the two group means. These results are consistent with Bargh and Schul's (1980) initial hypothesis that individuals must develop a good understanding of the problem and then structure that problem in a way that can be presented to others. Given the positive correlation between the quality of the concept map and problem solving achievement, the development of a good understanding of the problem may have been one critical factor for successful problem solving. It may also be the case that the concept maps served as a guideline for solving the problem. Indeed, students frequently referred back to the concept map when solving the problem (coordinating informational sources in our coding scheme) to assess which step they had completed and what they needed to solve next. Although there was no difference in the frequency with which students referred back to the concept map between groups, a higher quality concept map may have benefitted students in the teaching condition.

**Planning, goal setting, and cognitive strategies.** Given differences in task definitions between the two groups, we also expected that students in the teaching condition would engage in more planning and goal setting during learning compared with students in the control group. Specifically, we hypothesized that students in the teaching condition would plan to use more metacognitive strategies to ensure progress on understanding and completing the task, and set more goals for completing the problem compared with students in the control condition. Counter to our hypotheses, frequency of planning and goal setting did not differ between the two groups. This lack of difference is consistent with De Backer et al.'s (2012) study, wherein they found no differences in undergraduate students' use of planning strategies. In De Backer's study, planning occurred infrequently, whereas in our study, students engaged in nearly as much planning and goal setting as they did in use of cognitive and metacognitive strategies. We interpret

these results to suggest that perhaps due to the complexity of the problem, all students engaged in a high level of planning and goal setting to help them sort out the complexities of the problem (e.g., "I'll do this step first, and then try that one").

These results may also suggest that being given the task of teaching content to others does not affect the planning and goal setting stage of self-regulated learning during complex problem solving. We do not believe this to be the case. Rather, it could be that students set different kinds of goals or make different kinds of plans, depending on their learning condition. In this regard, frequency of planning and goal setting may not differ; differences may lie in the quality of plans and goals set. Future research should explore this possibility. Alternatively, perhaps due to the increase in the cognitive demands of the task, and the increase in metacognitive processes that occurred in the teaching group, students did not have the cognitive resources needed to be more planful during problem solving. Given that cognitive load was not measured during learning, our interpretations are highly speculative and require future research to explore these possibilities empirically. Additionally, it is important to note that power to detect a difference for this particular variable was less than optimal. However, we also note that frequency of planning and goal setting was actually slightly higher for learners in the control condition compared with the intervention condition. Certainly, larger sample sizes should be considered in future work in addition to an exploration of the types of plans that are made.

Similarly, we found no differences in the frequency of cognitive strategy use between the two learning conditions. From a theoretical perspective, this may suggest that the task of teaching others how to solve a complex problem does not change students' approaches to solving that problem, at least not for this kind of task. Alternatively, perhaps elementary students to not yet have a rich repertoire of deeper processing cognitive strategies that they could implement to understand and solve the complex problem (Butler & Winne, 1995; Zimmerman & Martinez-Pons, 1990). Rather, it may be the case that students in the teaching condition set higher standards for understanding and solving the problem, which would result in checking answers to ensure correct and complete solutions more frequently than students in the control condition. Unfortunately, we did not directly measure students' standards for problem solving. We suggest that future research explicitly ask students to indicate what their standards entail for solving the problem to assess whether differences arise between these learning conditions. It may be the case that a more fine-grained analysis of various components of self-regulated learning within each phase is necessary to paint a clearer empirical picture of the precise mechanisms involved.

**Metacognitive processes and mathematics achievement.** Previous research has also theorized that when learning by preparing to teach, individuals may engage in more metacognitive processes to ensure a deep understanding of the content to be able to later explain that content to others (Bargh & Schul, 1980; Fiorella & Mayer, 2013, 2014; Roscoe, 2014; Roscoe & Chi, 2007). Consistent with theoretical predictions, results from this study revealed that students in the learning by preparing to teach intervention engaged in substantially more metacognitive processes than students in the control condition (given the large effect size found for the difference between groups; see Lipsey et al., 2012). That is, the mean for metacognitive processes used for the

intervention group was higher than 71.90% of the scores in the control group; a difference that reflects 1.3 times more metacognitive strategy use in the intervention group compared with the control group.

As such, this study provides empirical evidence that the expectation of teaching changes the way individuals actually engage in learning at the metacognitive level (Bargh & Schul, 1980; Roscoe & Chi, 2007), which is essential for mathematics learning (de Corte et al., 2000; Muis, 2004; Schoenfeld, 1985; Zimmerman & Labuhn, 2012). Indeed, coupled with a better conceptualization of the problem, a higher frequency of metacognitive processes was predictive of better problem solving achievement. In fact, given the large effect size (see Lipsey et al., 2012) associated with the differences between the two groups on mathematics achievement, we infer that our intervention is very promising for improving students' learning processes and outcomes in this specific context. Indeed, the average achievement score for the intervention group was higher than 73.24% of the scores in the control group given the overall 7.73% difference between the group means. It may be that structuring a problem prior to solving it is critical for better problem solving outcomes within a learning by teaching paradigm.

This research also adds to the current literature on self-regulated learning. Learners in the learning by teaching group engaged in more frequent metacognitive processes, likely due to the differences in task definitions or the standard that students set for learning. As Muis (2007) suggests, differences in task definitions and standards for learning lead to differences in the way that individuals approach a learning task. Like previous research (Chevrier et al., 2015), when students are given different learning tasks, like preparing for a multiple-choice test that requires recognition of information to successfully complete the task, versus an inference verification task that requires learners to deeply understand the content to make correct inferences, the standards they set for learning differ, as do the self-regulatory strategies they use to prepare for those tasks. In our study, although we did not directly measure the standards that learners set for understanding and completing the problem, it may be the case that students in the teaching condition set higher standards, which resulted in more metacognitive processes.

Results from this research are also consistent with previous research (Zimmerman & Martinez-Pons, 1990) with regard to differences found between females and males in their use of learning strategies, and on mathematics problem solving performance (Hyde et al., 1990; Voyer & Voyer, 2014). Specifically, females used more learning strategies compared with males, and outperformed males on understanding the problem and on achievement outcomes. It is interesting to note that gains for females and males in the learning by preparing to teach condition were equivalent, which suggests that a gender bias was not at play in fostering improved learning outcomes. This has important educational implications.

## Educational Implications

From an instructional perspective, teachers can readily integrate this approach into their daily classrooms. Specifically, we developed our intervention with teachers who recently integrated tech-

nology into their classrooms, or were in the process of doing so. Although spending two hours to solve a complex problem may seem like a luxury, in the province of Quebec this is standard practice. Students must complete three provincially mandated situational problems each year, and teachers spend considerable classroom time having students practice these complex problems, spread out over several days. Practicing these complex problems within a learning by teaching paradigm may help to improve students' ability to solve these complex problems, which mirror the kinds of complex problems people face outside of the school context. Moreover, students as young as 10 to 11 years of age were capable of using the various tablet applications to create concept maps of the problem, solve the problem, and subsequently develop a teaching video. With use of videos, teachers can further assess students' understanding of the problem, and identify gaps in students' understanding or misconceptions that can be subsequently addressed. We speculate that further learning gains can be obtained from actually developing the video, and it is likely the case that students continue to monitor and correct errors during the second phase of learning by teaching. Future research that we plan to conduct will assess this possibility.

## Recommendations for Future Research

Further research is also needed to test the additional effects of actually teaching on student learning. As discussed earlier, four lines of research have been conducted within the learning by teaching paradigm, including learning by preparing to teach (e.g., Annis, 1983; Bargh & Schul, 1980; Fiorella & Mayer, 2013; Renkl, 1995), learning by teaching (Annis, 1983; Fiorella & Mayer, 2013, 2014), learning through peer tutoring (Chi et al., 2001; Roscoe, 2014; Roscoe & Chi, 2007), and teachable agents (Biswas et al., 2005, 2010; Roscoe et al., 2013). Although this study only focused on learning by preparing to teach, other studies in the learning by teaching literature have systematically explored the effects of learning by preparing to teach versus actually teaching (Annis, 1983; Fiorella & Mayer, 2013, 2014). Actual teaching includes both components of teaching which are preparing to teach and explaining content to others. These studies found that actually teaching provided additional benefits with regard to learning gains, but more work is necessary to systemically explore why these gains are achieved. Future research should include a learning by teaching group, which will allow students to develop additional teachable self-regulatory skills and an even deeper understanding of the content (Fiorella & Mayer, 2013, 2014). This is important because previous research has shown that elementary school students are poor self-regulated learners (Schoenfeld, 1994; Schunk & Zimmerman, 1997).

Future studies should also assess whether there are motivational and emotional implications in the learning by teaching condition. For example, according to Pekrun's (2006) control-value theory of achievement emotions, high value for a specific task and high perceived control will lead to enjoyment and increased learning (Johnson & Sinatra, 2013; Pekrun, 2006). On the other hand, low task value and low control will lead to boredom and decreased learning (Pekrun, 2006). Thus, future studies should assess students' emotions during learning when preparing to teach, and assess whether they value learning more when they are expected to teach the content. If the expectation



of teaching increases value, this may have additional motivational benefits (Pekrun, 2006).

### Conclusion

The purpose of this research was to explore whether learners' task definitions and self-regulatory processes differed when learning by teaching versus learning for learning during complex mathematics problem solving. Previous research has focused on the learning outcomes of learning by teaching. This research advances understanding of how learning by preparing to teach improves learning outcomes by exploring the specific mechanisms involved. These results provide empirical evidence of theoretical predictions that learners who learn by teaching engage in more frequent metacognitive processes, likely due to the differences in task definitions or standards students set for learning (Muis, 2007). Given the authentic nature of our research, carried out in actual classrooms in collaboration with teachers, we believe that this work has important education implications. With the availability of technology increasing at a rapid rate, it is imperative that interventions are developed and empirically assessed to reap the benefits that technology may have to offer.

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## Appendix A

### Quebec Exam in Mathematics (2009) “Start Your Engines” Marking Rubric

|   | Your Mark | Total Mark |
|---|-----------|------------|
| <b>Racetrack design:</b>  |           |            |
| 7-sided polygon   |           | 7          |
| Perimeter between 4.5 km and 5 km   |           | 4          |
| Measures of each line segment (with ruler & label)  |           | 4          |
| 1 acute angle, 1 obtuse angle, and 1 reflex [included and correctly labeled]  |           | 6          |
| Identifies the starting line with an “S”  |           | 1          |
| <b>Spectator area:</b>  |           |            |
| 8 sections  |           | 1          |
| Letter identification for each section (A, B, etc.)   |           | 1          |
| 15 squares per section  |           | 1          |
| <b>Starting line frieze pattern:</b>  |           |            |
| Rectangular design measuring 6 squares by 3 squares, reflected twice  |           | 3          |
| 1/3 white and 2/3 black   |           | 3          |
| <b>Cost of starting line painting:</b>  |           |            |
| Costs \$112.50  |           | 1          |
| <b>Calculations (example):</b>  |           |            |
| 50 cm represents <b>5,000 m</b>   |           | 3          |
| $6\text{ cm} + 5\text{ cm} + 4\text{ cm} + 9\text{ cm} + 5\text{ cm} + 10\text{ cm} + 10\text{ cm} = \mathbf{49\text{ cm}}$ |           | 3          |
| $15,000 \div 1,000 = \mathbf{15}$ squares   |           | 3          |
| $120,000 \div 15,000 = \mathbf{8}$ sections   |           | 3          |
| 6 squares white, 12 squares black, 18 squares total   |           | 3          |
| $\$6.25/\text{m}^2 \times 18\text{ m}^2 = \$112.50$   |           | 3          |
| <b>Total:</b>   |           | <b>50</b>  |
| <b>Percent:</b>   |           |            |

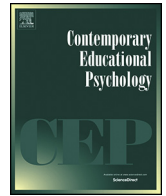
(Appendices continue)

## Appendix B

### Concept Map Marking Rubric

|   | Your Mark | Total Mark   |
|---|-----------|--------------|
| <b>Blue Border Start Your Engines Title [1]</b>                             |           | <b>1</b>     |
| <b>Red Border (multiple answers 1 out of 6) [1]</b>                         |           | <b>1</b>     |
| 7-sided, perimeter, angles, # sections, # squares spectator, # squares line |           |              |
| <i>Please write red choice:</i> _____                                       |           |              |
| <b>Racetrack design: [6]</b>  |           |              |
| <b>Green borders</b>  |           |              |
| Centimeters to meters conversion  |           | <b>1</b>     |
| Perimeter addition  |           | <b>1</b>     |
| <b>Black borders</b>  |           |              |
| Identifies the starting line with an "S"                                    |           | <b>1</b>     |
| 7-sided polygon   |           | <b>1</b>     |
| Perimeter between 4.5 km and 5 km   |           | <b>1</b>     |
| At least 1 acute angle, 1 obtuse angle, and 1 reflex                        |           | <b>1</b>     |
| <b>Spectator area: [5]</b>  |           |              |
| <b>Green borders</b>  |           |              |
| Number of sections  |           | <b>1</b>     |
| Number squares per section  |           | <b>1</b>     |
| <b>Black borders</b>  |           |              |
| Total: 120,000 people   |           | <b>1</b>     |
| 15,000 people per section   |           | <b>1</b>     |
| Letter identification for each section                                      |           | <b>1</b>     |
| <b>Starting line frieze pattern: [4]</b>                                    |           |              |
| <b>Green borders</b>  |           |              |
| Total number squares starting line  |           | <b>1</b>     |
| Number white squares starting line  |           | <b>1</b>     |
| <b>Black borders</b>  |           |              |
| Reflected twice   |           | <b>1</b>     |
| One third white   |           | <b>1</b>     |
| <b>Cost of starting line painting: [3]</b>                                  |           |              |
| <b>Green borders</b>  |           |              |
| Cost of paint   |           | <b>1</b>     |
| <b>Black borders</b>  |           |              |
| \$6.25 per square meter   |           | <b>1</b>     |
| Starting line 18 m by 3 m   |           | <b>1</b>     |
| <b>Subtotal:</b>  |           | <b>20</b>    |
| <i>−0.5 for each bubble wrong color (___ × −0.5)</i>                        |           | <i>−</i> ___ |
| <i>+0.5 for correct links between concepts</i> _____                        |           | <i>+</i> ___ |
| <b>Final New Total:</b>   |           | <b>20</b>    |
| <b>Percent:</b>   |           |              |

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# The role of epistemic emotions in mathematics problem solving

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## ABSTRACT

The purpose of this research was to examine the antecedents and consequences of epistemic and activity emotions in the context of complex mathematics problem solving. Seventy-nine elementary students from the fifth grade participated. Students self-reported their perceptions of control and value specific to mathematics problem solving, and were given a complex mathematics problem to solve over a period of several days. At specific time intervals during problem solving, students reported their epistemic and activity emotions. To capture self-regulatory processes, students thought out loud as they solved the problem. Path analyses revealed that both perceived control and value served as important antecedents to the epistemic and activity emotions students experienced during problem solving. Epistemic and activity emotions also predicted the types of processing strategies students used across three phases of self-regulated learning during problem solving. Finally, shallow and deep processing cognitive and metacognitive strategies positively predicted problem-solving performance. Theoretical and educational implications are discussed.

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## 1. Introduction

“Mathematics” and “anxiety” are two words that often go together when individuals are asked to think about their experiences when learning mathematics. Indeed, a rich literature on test anxiety (see Zeidner, 1998) demonstrates researchers’ interest in this particular topic over the past five decades (Schutz & Pekrun, 2007). Research on test anxiety has explored the structure, antecedents, and effects that anxiety has on students’ health and well-being, and on learning outcomes in general. Despite the prominence of test anxiety research, with the exception of causal attributions as antecedents to achievement emotions (Weiner, 1985), only recently have educational psychologists considered the role that various types of emotions play in educational contexts. Today, emotions are recognized as being critically important to students’ learning, motivation, and academic achievement as well as teachers’ productivity (Efklides & Volet, 2005; Linnenbrink, 2006; Schutz & Pekrun, 2007).

In contemporary educational research, theorists define emotions as multifaceted phenomena that involve cognitive, affective,

physiological, motivational, and expressive processes (Scherer, 2000). For example, the anxiety a student experiences about a mathematics exam may consist of worrying about failing the exam (cognitive), feelings of nervousness (affective), increased cardiovascular activation (physiological), impulses to flee the situation (motivational), and anxious facial expression (expressive) (Pekrun & Stephens, 2012). Over the past ten years, research has focused on what kinds of emotions are experienced in educational settings, as well as how both positive and negative emotions relate to achievement and personal growth (Efklides & Volet, 2005; Linnenbrink, 2006; Linnenbrink-Garcia & Pekrun, 2011; Pekrun, Goetz, Titz, & Perry, 2002a; Schutz & Lanehart, 2002). Research has shown that positive emotional experiences relate to students’ academic achievement and success in an academic domain (Pekrun, Elliot, & Maier, 2009), whereas the converse is found for negative emotional experiences (Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011). Emotions such as enjoyment, hope, and pride positively predict academic achievement, whereas negative emotions like boredom and hopelessness can lead to a decrease in achievement (Pekrun et al., 2011). Additionally, both positive and negative emotions play an important role in self-regulated learning, strategy use, and motivation (Op’t Eynde, De Corte, & Verschaffel, 2007; Pekrun, Goetz, Titz, & Perry, 2002b).

Given the role that emotions play in learning and achievement, theoretical models have been developed to describe both the antecedents of students’ emotional experiences, as well as their consequences. For example, Pekrun and colleagues (Pekrun, 2000, 2006; Pekrun, Frenzel, Goetz, & Perry, 2007) proposed the control-value theory of achievement emotions to explore the antecedents and consequences of emotions experienced in academic settings.

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Achievement emotions are defined as emotions that are linked to achievement activities or achievement outcomes. Activity emotions are emotions experienced during engagement in an activity (e.g., solving a mathematics problem), whereas outcome emotions include both prospective outcome emotions (e.g., related to possible successes or failures) and retrospective outcome emotions (e.g., linked to prior success and failure). Achievement, activity, and outcome emotions fall under the broader category of academic emotions.

Recently, theorists have expanded the range of emotions to include epistemic emotions which, following R. Pekrun's suggestion (personal communication, June 4, 2015), we define as emotions that arise when the object of their focus is on knowledge and knowing (see also Pekrun & Linnenbrink-Garcia, 2012; Pekrun & Stephens, 2012). The word *epistemic* refers specifically to facets of knowledge and knowing. Like epistemic beliefs, individuals' beliefs about knowledge and knowing (Hofer & Pintrich, 1997), and epistemic cognition, cognitive manifestations of individuals' epistemic beliefs in context (Muis, Trevors, & Chevrier, *in press*), the object focus for epistemic emotions is on knowledge and processes of knowing. Typical examples include surprise, curiosity, and confusion that arise when there is unexpected information or cognitive incongruity (Kang et al., 2009). Cognitive incongruity might entail conflicting information (e.g., a student recalculates her solution but derives two different answers), or information that is counter to what one believes to be true (e.g., a student believes an acute angle is greater than 180 degrees but is told that it is less than 180 degrees). When individuals experience conflicting information, their first reaction may be surprise. Individuals may then experience curiosity about the conflicting information and attempt to resolve it, or they may experience confusion if the incongruence cannot be resolved.

Philosophers have also considered the role that epistemic emotions play during knowledge acquisition (Brun, Doğuoğlu, & Kuenzle, 2008; Morton, 2010), and have focused primarily on three: curiosity, surprise, and confusion. From an epistemological standpoint, these three affective states represent epistemic emotions because they relate to the knowledge-generating aspects of tasks and activities (see Brun et al., 2008 and Morton, 2010 for overviews). As philosophers have argued, they represent a major category of human emotion that serves an evolutionary-based purpose of acquiring knowledge about the world and the self (Brun et al., 2008). Brun and Kuenzle (2008) differentiate epistemic emotions from other types of emotions such as social, moral, or achievement emotions in terms of their specific object focus. For epistemic emotions, the object of the emotion is knowledge and knowledge generation. In contrast, for social, moral, or achievement emotions, other individuals, moral norms, or success and failure, respectively, are their object focus.

Additionally, as Brun and Kuenzle (2008) suggest, surprise, curiosity, and confusion are epistemic by their very nature, whereas other emotions can belong to different categories of emotions depending on their object focus. For example, frustration at not deriving a correct solution to a mathematics problem may be regarded as an epistemic emotion if the focus is on the cognitive incongruity that resulted from the unsolved problem. If, however, the focus is on personal failure and the inability to solve the problem, then the emotion is considered an achievement emotion. However, since epistemic emotions occur during learning and pertain to the features of ongoing knowledge-generating activities, like achievement emotions, epistemic emotions can be considered activity emotions. As such, epistemic emotions should share similar features to activity emotions with regard to their role in self-regulated learning, which is defined as "learning that results from students' self-generated thoughts and behaviors that are systematically oriented toward the attainment of their learning goals" (Schunk, 2001, p. 125).

For example, Morton (2010) delineates how curiosity is a driving force behind how individuals approach solving a complex problem. When individuals are curious about answers to complex prob-

lems, this influences how they plan to solve the problem, which goals they set, and the strategies they use to achieve their goals. When confusion arises, individuals will attempt to resolve the confusion by evaluating the source of confusion, adjusting strategies, and monitoring whether the confusion has been resolved. Planning, goal setting, strategy use, and metacognitive monitoring and control are all key features of models of self-regulating within the educational psychology literature (Puustinen & Pulkkinen, 2001). In this regard, similar to Pekrun's (2006) delineation of the role of activity emotions in self-regulated learning, we hypothesize that epistemic emotions should also be related to various phases of self-regulated learning. Indeed, recent empirical work supports this contention.

Specifically, research on epistemic emotions has shown that curiosity positively predicts the use of deep processing cognitive and metacognitive strategies, including metacognitive monitoring and evaluation of learning, as well as critical thinking and elaboration of content, whereas surprise negatively predicts critical thinking (Muis et al., *accepted*). D'Mello, Lehman, Pekrun, and Graesser (2014) found that confusion is beneficial for learning when that confusion can be resolved through the use of appropriate learning strategies. Despite these promising new lines of research, very little is known with regard to the antecedents and consequences of epistemic emotions, particularly with younger students. Moreover, with regard to self-regulated learning, the majority of research on achievement emotions has focused solely on the learning strategies that learners adopt during the enactment phase of self-regulated learning. To better understand the role that emotions play in self-regulated learning, a logical next step in this line of research is to examine whether emotions predict processes that occur across several phases of self-regulated learning (Muis, 2007). Our research addresses these gaps in the literature by extending research on academic emotions and their link to theoretical models of self-regulated learning. Specifically, the purpose of our research was to examine the role that epistemic emotions play in complex mathematics problem solving during various phases of self-regulated learning with a sample of elementary students. Prior to delineating our research questions and hypotheses, we first describe relevant theoretical frameworks and empirical work. We begin with Pekrun's (2006) control-value theory of achievement emotions.

### 1.1. Pekrun's (2006) control-value theory of achievement emotions

Pekrun (2006) proposed that the types of emotions individuals experience in an achievement setting depend on their perceptions of control (both action control and outcome control) as well as their value appraisals, both of which arise as a function of the environment (e.g., cognitive quality, motivational quality, goal structures, et cetera). That is, both control and value are considered important antecedents to the kinds of emotions individuals experience during an achievement situation. Control appraisals refer to the perceived controllability of the achievement-related actions and outcomes, whereas value refers to the subjective importance of the achievement-related activities and outcomes and include both intrinsic and extrinsic value (Pekrun et al., 2011). Perceptions of control and value are theorized to initiate different kinds of achievement emotions, both prospectively and retrospectively, as well as during learning. For example, for activity emotions (e.g., experienced during engagement in a learning task), when individuals perceive high levels of control and value for learning mathematics, they will experience enjoyment during mathematics problem solving. However, anger and frustration may arise during complex problem solving if individuals do not place much value on mathematics and the task demands are high, or they may experience anxiety when perceived value is high but control is low. Finally, boredom is experienced when individuals perceive low control and low value in contexts under which individuals are over-challenged.



### 1.1.1. Epistemic emotions

According to Graesser, Ozuru, and Sullins (2010), surprise, curiosity, and confusion are very likely to arise during complex learning tasks. For example, in cases of complex mathematics problem solving, individuals must first attempt to understand the problem, generate relevant prior knowledge, coordinate informational sources, make comparisons, and generate inferences to correctly solve the problem (Schoenfeld, 1985). At any point during these processes, discrepant events may arise that induce cognitive incongruity, which can entail obstacles to goals, impasses, or unexpected feedback such as an incorrect answer (Graesser, Lu, Olde, Cooper-Pye, & Whitten, 2005; VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003). Indeed, within the mathematics education literature, researchers have documented the importance of productive struggle to facilitate students' understanding of mathematics (Brown, 1993; Hiebert & Grouws, 2007; Kapur, 2008; Kapur & Bielaczyc, 2012). They describe productive struggle as occurring in situations when students expend effort to make sense of a mathematics problem when the solution is not immediately apparent. Confusion arises when solving a problem that is within students' reach, but does not lead to extreme levels of frustration that may arise when a problem is too challenging. When confusion arises, students attempt to resolve this confusion to reduce cognitive dissonance (Festinger, 1957). As Brown (1993) argued, confusion is viewed as facilitating the process of understanding given that students must make mental connections among mathematical facts, ideas, and procedures to resolve the confusion to subsequently solve the problem.

As such, given the likely prevalence of these emotions during complex mathematics problem solving, research is needed to further explore under which conditions these emotions likely arise, and to establish the antecedents and consequences of these emotions when they do arise. Results from this initial foray into the role that epistemic emotions play during complex learning can then be used to develop learning environments that foster better learning outcomes.

### 1.1.2. Antecedents and consequences

What might be the antecedents and consequences of epistemic emotions? Drawing from the philosophical literature, Morton (2010) suggests that epistemic emotions arise when individuals attempt to acquire accurate beliefs (in the philosophical sense of a justified true belief). Individuals are curious when the answers to questions have practical importance to them. Certainly, individuals can engage in problem solving without being curious, but curiosity drives deeper engagement during problem solving. Take, for example, a problem that has important implications for the safety of many individuals (e.g., correctly designing a bridge that connects two cities), and several factors must be taken into consideration when solving the problem (e.g., wind, earthquakes, traffic volume). If it is important to the individual that a correct answer is achieved, that individual will be driven by curiosity to correctly solve the problem. In this situation, the individual is more likely to be vigilant while solving the problem, ensuring to understand the problem and its various components, generating relevant prior knowledge, coordinating informational sources, making comparisons, and generating inferences, which are considered deep processing strategies. The individual is also more likely to continually evaluate progress, and check answers against set goals, which are key metacognitive processes for successful problem solving (Jacobse & Harskamp, 2012). The importance placed on correctly solving the problem may also result in anxiety if issues arise when solving the problem, confusion or frustration if the issue cannot be resolved, or enjoyment if there is resolution. Moreover, if high value is placed on mathematics problem solving, but individuals perceive low control, they are more likely to experience higher levels

of anxiety. Finally, given that surprise is considered a neutral emotion that is triggered in response to events during the learning task, neither control nor value is hypothesized to be antecedents to this emotion (surprise is considered a neutral activating emotion (Mauss & Robinson, 2009)).

With regard to consequences, a number of empirical studies have explored the role that emotions play during learning, and have found that emotions influence a wide range of cognitive and metacognitive processes, including attention, perception, social judgment, cognitive problem solving, decision making, and memory processes (see Pekrun & Linnenbrink-Garcia (2012) for a review). Given the relationship that researchers have found between emotions and learning strategies, Pekrun (2006) proposed that emotions predict the types of cognitive and metacognitive strategies individuals use during learning. Specifically, Pekrun proposed that positive activating emotions (curiosity, enjoyment) result in deep processing learning strategies, including generating inferences and metacognitive strategies (see Pekrun & Stephens, 2012, for a review of supporting evidence). In contrast, negative activating emotions (anxiety, frustration) result in shallow processing strategies such as rereading (Pekrun & Stephens, 2012), whereas confusion and surprise result in an increase in metacognitive strategies to reduce cognitive incongruity (Pekrun & Stephens, 2012). Negative deactivating emotions, such as boredom, can impair systematic use of learning strategies (Pekrun, Goetz, Daniels, Stupnisky, & Perry, 2010) and therefore result in reduced cognitive and metacognitive strategies. Finally, from a self-regulatory perspective (e.g., Muis, 2007), and drawing from Pekrun's (2006) model, use of deep learning strategies is theorized to have positive effects on learning outcomes, particularly with complex learning material. Several empirical studies support these hypotheses (e.g., Azevedo & Chauncey Strain, 2011; Azevedo et al., 2013; Murayama, Pekrun, Lichtenfeld, & vom Hofe, 2013; Pekrun et al., 2010, 2011).

### 1.1.3. Extending theoretical considerations

We further posit that the consequences of academic emotions need not be limited to the enactment phase of self-regulated learning. That is, as Muis (2007) proposed, emotions are activated during the task definition phase (the first phase) of self-regulated learning. Similar to most models of self-regulated learning (Puustinen & Pulkkinen, 2001), Muis (2007) proposed four phases of learning: 1) task definition, 2) planning and goal setting, 3) enactment, and 4) evaluation. In the first phase of learning, an individual constructs a perception of the task, which is influenced by external conditions, such as context, and internal conditions, such as prior knowledge, motivation, and emotions. During the second phase, components from the first phase influence the types of goals an individual sets for learning and the plans made for carrying out the task. The third phase begins when an individual carries out the task by enacting the chosen learning strategies. In the last phase, individuals evaluate the successes or failures of each phase or products created for the task, or perceptions about the self or context. Products created during learning are compared to the standards set via metacognitive monitoring. Key to the evaluation phase is metacognition, but metacognitive processes can occur during any phase of self-regulated learning, and individuals may cycle through any phase as learning progresses. This reflects the cyclical nature of her model. Accordingly, given that emotions are activated during the task definition phase of learning, we propose that emotions predict the plans and goals that students set for learning, the learning strategies students use (enactment phase), as well as the metacognitive processes used to monitor progress and evaluate products (evaluation phase) created during learning. In the next section, we focus specifically on empirical evidence of the consequences of epistemic emotions.

## 1.2. Empirical evidence of epistemic emotions

With a specific focus on epistemic emotions (e.g., curiosity, surprise, confusion), a few studies have explored the role they play in complex learning (e.g., Craig, Graesser, Sullins, & Gholson, 2004; D'Mello & Graesser, 2011; Graesser, Chipman, King, McDaniel, & D'Mello, 2007; Muis et al., *accepted*). For example, D'Mello et al. (2014) tested a theoretical model, which posits that confusion, triggered by cognitive conflict, can be beneficial for learning if appropriately induced, regulated and resolved. Confusion was experimentally induced via an animated agent that presented contradictory information, and participants were required to decide which opinion had more scientific merit. Results revealed that contradictions had no effect on learning when learners were not confused by the manipulations. However, when confusion did arise, participants' performance on multiple-choice and transfer tests was substantially higher than the control (no contradiction) condition.

Based on these results, D'Mello et al. (2014) argued that confusion is beneficial for learning when individuals are driven by the need to reduce that confusion. That is, once an impasse is detected, learners may engage in more effortful learning strategies to resolve the confusion (enactment phase), such as careful deliberation of the situation, evaluation of progress made (evaluation phase), or reconsideration of the problem space (task re-definition phase). However, D'Mello et al. (2014) also warned that not all confusion leads to greater learning gains. In the context of mathematics problem solving, a learner may become confused about the problem, make several unsuccessful attempts to resolve the issue, but not succeed. In this case, the individual may resort to shallow processing strategies given the limited cognitive resources available, which may then result in little learning gains.

In another study, D'Mello, Lehman, and Person (2010) measured the emotions students experienced during a series of effortful problem solving activities, and assessed whether various emotions predicted achievement outcomes. Forty-one undergraduate students solved challenging analytic reasoning problems, and emotions were measured at random and specific times throughout the problem-solving session. Results revealed the primary emotions that students experienced included curiosity, happiness, confusion, frustration, boredom, and anxiety. Moreover, curiosity was a positive predictor of problem solving performance, whereas frustration was a negative predictor of performance.

Given this set of studies, and those within the mathematics education literature (e.g., Kapur, 2008; Kapur & Kinzer, 2009), it appears that curiosity and confusion can be beneficial for complex learning tasks by driving deep processing cognitive and metacognitive strategies to resolve cognitive conflict that arises during learning. What is still unknown, however, is whether these relations extend to other phases of self-regulated learning and whether similar patterns would result with younger elementary students in authentic learning contexts. Specifically, to date, research on the role of confusion has been conducted solely with adolescent and adult samples. Given the important function of emotions in authentic educational contexts (see Pekrun, 2006), research is needed to assess the role epistemic emotions play during complex learning with younger students. As Butler and Winne (1995) and Zimmerman and Martinez-Pons (1990) noted, younger students are not very good at self-regulating their learning, nor are they necessarily accurate at judging how well they are carrying out a task. As such, in the face of an impasse during complex problem solving, it could be the case that younger students do not adjust their strategies, redefine the problem space, or implement deeper processing cognitive and metacognitive strategies to resolve conflict. Confusion that arises may result in the reduction of processing strategies altogether. Moreover, to date, research has not empirically investigated possible antecedents to curiosity, surprise and confusion, such as control and

value, or whether confusion arises due to cognitive conflict regardless of students' perceptions of control and value for mathematics problem solving. We addressed these gaps in the literature.

## 2. The current study

As previously noted, during complex problem solving, confusion is likely to arise given that mathematics is inevitably coupled with making mistakes and recovering from those mistakes. Students may experience confusion when mistakes arise, or when they are not able to successfully solve one or more aspects of a complex problem. Curiosity may drive them to persist in the face of difficulty, to use more deep processing cognitive and metacognitive strategies to resolve issues, and ultimately successfully solving the problem. Alternatively, students may experience anxiety, frustration, and boredom when issues are not resolved, which may negatively affect subsequent learning processes and learning outcomes. As such, it is paramount for researchers to assess what the antecedents and consequences are with regard to epistemic emotions, and to academic emotions more generally, during complex mathematics problem solving.

Specifically, following Pekrun's (2006) control-value theory of achievement emotions, we sought to explore whether perceived value (intrinsic interest value, importance, and utility value) and control during mathematics problem solving were antecedents to students' epistemic and activity emotions during mathematics problem solving, and whether emotions predicted planning and goal setting (Phase 2), and actual use of shallow and deep cognitive strategies (Phase 3), as well as deep metacognitive strategies (Phase 4). We further explored whether learning processes mediated relations between control and value and achievement, or between emotions and achievement. Seventy-nine fifth grade elementary students participated, and were given a complex mathematics problem to solve over a period of several days. We targeted grade five students given that the provincially mandated mathematics curriculum includes multi-faceted complex mathematics problems that all students are required to complete and that count as a percentage of their final grades (30%).

We addressed the following research questions: (1) Are perceived control and value antecedents to students' epistemic and activity emotions? (2) Do epistemic/activity emotions mediate relations between control and value and achievement? (3) What is the relationship between epistemic/activity emotions and learning processes across three of the four phases of self-regulated learning during complex mathematics problem solving? (4) Do learning processes mediate relations between emotions and achievement? (5) Are learning processes predictors of mathematics problem solving achievement? Based on theoretical (Morton, 2010; Pekrun, 2006) and empirical considerations, we hypothesize that both perceived value and control will be significant predictors of students' epistemic and activity emotions. Specifically, value and control will both positively predict curiosity and negatively predict confusion. Additionally, coupled with low levels of control, higher value will predict higher levels of anxiety and frustration, whereas both high control and high value will predict enjoyment. In contrast, low levels of control and value will predict boredom. Finally, greater perceived control will predict higher levels of curiosity and lower levels of confusion. We further hypothesize that emotions would predict planning and goal setting (Phase 2), cognitive strategies employed during the enactment phase of self-regulated learning (Phase 3), as well as metacognitive strategies employed during the evaluation phase (Phase 4). Specifically, higher levels of curiosity, enjoyment, and confusion will positively predict use of plans and goals (or adjustments to plans and goals, given the cyclical nature of self-regulated learning), and deeper cognitive and metacognitive strategies, whereas surprise, anxiety and frustration will positively predict shallow

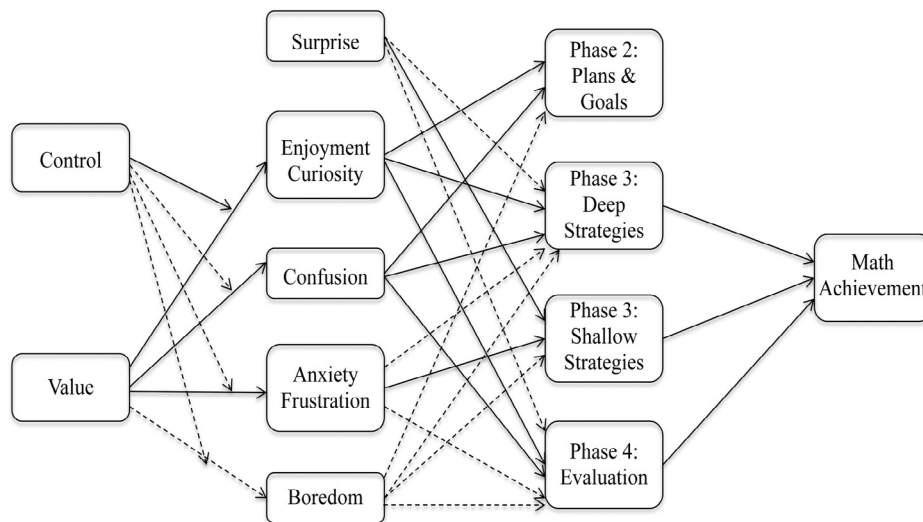


Fig. 1. Hypothesized model. Solid lines indicate positive relationships, whereas dotted lines denote negative relationships.

processing strategies and negatively predict deep processing cognitive and metacognitive strategies. Given that boredom results in disengagement in learning (Linnenbrink-Garcia & Pekrun, 2011), we hypothesize that boredom would negatively predict use of planning and goal setting, and cognitive and metacognitive learning strategies. Finally, we predict that deep processing cognitive and metacognitive strategies will result in higher levels of problem solving achievement, and will mediate relations between emotions and achievement. Our hypothesized model is presented in Fig. 1.

### 3. Methodology

#### 3.1. Participants

Seventy-nine fifth-grade students ( $n = 34$  females) from two different schools across four classrooms participated. All students were from the same school board. These two schools were chosen given their eclectic mix of low- through high-income families within each school and inclusion of approximately 30% of students on individualized education plans in each classroom. Variability in student characteristics allowed for a broader generalization of the results. There were 41 students ( $n = 20$  females) from one school, and 38 ( $n = 14$  females) students from the other school. The mean age of the sample was 11 years ( $SD = .31$ ). All grade 5 students in both schools were invited to participate and 95% assented to participate (parental consent was also obtained).

#### 3.2. Materials

##### 3.2.1. Prior knowledge

Students' standardized achievement score on the 2013 compulsory provincial exam was used to obtain a measure of prior knowledge. The 2013 provincial exam was completed one week prior to the beginning of the research study (in the first week of April, which is the eighth month of the school year). Commencement of the research study was intentionally chosen to immediately follow the provincial exam to ensure a valid assessment of students' prior knowledge. The exam included a series of multiple-choice questions that assessed students' knowledge of the mathematics content

covered over the school year. Reliability of the prior knowledge test was .94.

##### 3.2.2. Global emotions about mathematics

To measure students' global emotions about mathematics, we used Pekrun, Lichtenfeld, Killi, and Reiss's (2007) Achievement Emotions Questionnaire (AEQ)—Elementary Version, which assesses students' enjoyment, boredom, and anxiety for mathematics class (12 items, e.g., "I enjoy math class"), mathematics homework (eight items, e.g., "Math homework bores me to death"), and mathematics tests (eight items, e.g., "I get very nervous during math tests"). Students rated each item on a 5-point scale ranging from "Not at all" (a rating of 1) to "very much" (a rating of 5). These emotions were used as a baseline measure of students' general emotions about mathematics prior to solving the problem and to ensure equivalence across schools. Cronbach's alpha reliability estimates for the three subscales were acceptable: .92 for enjoyment, .90 for boredom, and .72 for anxiety in class; .79 for enjoyment, .85 for boredom, and .70 for anxiety for homework; and, .86 for enjoyment and .88 for anxiety during tests.

##### 3.2.3. Task value

Pekrun and Meier's (2011) Task Value Measure (adapted from Eccles, Wigfield, Harold, & Blumenfeld, 1993) was used to measure students' value for learning mathematics in general, as well as their perceptions specifically for mathematics problem solving. This seven-item Likert scale measures three dimensions of task value: intrinsic interest value (two items, e.g., "In general, I find learning about math very interesting"), importance (two items, e.g., "Learning more about math is very important"), and utility value (three items, e.g., "In general, learning about math is useful"). At four different time points, students rated each item on a 5-point scale ranging from "Not at all true of me" (a rating of 1) to "Very true of me" (a rating of 5). The first time (general context) was done two weeks prior to being given the mathematics problem. Students were instructed to think about mathematics in general (again, to assess similarity across schools). The second and subsequent times were conducted during each problem solving session. Students were instructed to respond to items based on their immediate experience of solving the mathematics problem. Because previous research has shown that younger students do not differentiate between the three types of value (see



Wigfield, 1994 for a complete review), all items were summed and averaged for an overall estimate of students' task<sup>1</sup> value for both general mathematics learning and specific to the problem-solving context for each of the three days. Higher values represent higher perceptions of task value. Cronbach's alpha reliability estimates were .88 for the general responses, and .88, .84, and .86 for each of the three days of mathematics problem solving specifically.

### 3.2.4. Academic control

To measure their perceived control (both action and outcome) for learning mathematics in general as well as specifically for mathematics problem solving, students completed Perry, Hladkyi, Pekrun, and Pelletier's (2001) Academic Control Scale, modified for elementary students, over four sessions. For the first session, two weeks prior to being given the problem to solve, students rated their level of agreement to each of the eight items, ranging from "Strongly disagree" (a rating of 1) to "Strongly agree" (a rating of 5). Sample items included, "I have a lot of control over my grades in math" and "The more effort I put into learning math, the better I do." Instructions for the first self-report session focused on mathematics in general. For the second and subsequent sessions, completed immediately following each day of problem solving, students were instructed to complete the scale with a specific focus on their perceptions of control during the problem solving session. Following previous research (Muis et al., accepted), all items were summed and averaged for an overall estimate of students' perceived control prior to (general context) and during problem solving (specific context). Higher values represent higher perceptions of control. Cronbach's alpha reliability estimates were acceptable at .75 for the general responses, and .71, .78, and .78 for each of the three days with a specific focus on mathematics problem solving.

### 3.2.5. Epistemic and activity emotions

The epistemic and activity emotions students experienced while solving the complex mathematics problem were measured using the Epistemic Emotions Scale (EES; Pekrun & Meier, 2011), adapted for elementary students (four items were removed from the original scale as elementary students would not likely understand their meaning, e.g., "muddled"). This 17-item self-report questionnaire is designed to measure three epistemic emotions and four activity emotions including curiosity (two items; e.g., interested), surprise (two items; e.g., shocked), confusion (two items; e.g., puzzled), and enjoyment (three items; e.g., joyful), anxiety (three items; e.g., nervous), frustration (two items; e.g., irritated), and boredom (two items; e.g., dull). Each item consisted of a single word describing one emotion (e.g., "excited"). Students were instructed to report the emotions they experienced when solving the mathematics problem. To assess emotions as they occurred during problem solving, students completed the scale at defined intervals across the three days of problem solving. All students completed the scales at the same time intervals (e.g., 10 minutes into problem solving, followed by 20, 30, 60 and 90 minutes). Students were asked to rate along a 5-point Likert scale how strongly they felt each of the emotions. Responses ranged from "Not at all" (a rating of 1) to "Very strong" (a rating of 5). Cronbach's alpha reliability estimates were within an acceptable range across each of the days, from .78 to .96. Specific values were as follows: surprise (.78 day one, .80 day two, and .79 for day three), curiosity (.87 day one, .83 day two, and .89 day three), enjoyment (.82 day one, .90 day two, .96 for day three), anxiety (.94 day 1, .89 day two, .78 day three), frustration (.80 day one, .84 day

two, .79 day three), and boredom (.85 day one, .86 day two, .84 day three).

### 3.2.6. Situational problem

The situational problem, *Start Your Engines*, was drawn from the 2009 compulsory Quebec Exam in Mathematics. The objective is to have students develop a coherent solution to a situational problem that meets the following conditions: (1) the procedure required to solve the situational problem is not obvious, since it involves choosing a significant number of previously acquired mathematical concepts and processes and using them in a new way; (2) the situation focuses on obstacles to overcome, which requires various learning strategies; and, (3) the instructions do not suggest a procedure to be followed or the mathematical concepts and processes to be used (Ministère de l'Éducation, des Loisirs et du Sport, 2009). For this particular problem, students had to: create a seven-sided polygon for the racetrack design that ranged in length between 4.5 km and 5 km; include at least one acute angle, one obtuse angle, and one angle greater than 180 degrees; create spectator areas with 15 squares per section to seat 120,000 spectators; draw a starting line frieze pattern that was one-third white, reflected twice; and, calculate the cost of the paint for the starting line.

### 3.3. Procedure

Parental consent and student assent were obtained, which included permission to participate in the study as well as permission to audio-record students' thought processes. Basic demographic data were also collected, which included students' gender, age, first language and other languages spoken at home. Following this, one week prior to giving students the complex mathematics problem, students completed the AEQ—Elementary Version (Pekrun, Lichtenfeld et al., 2007), followed by the Task Value Measure (Pekrun & Meier, 2011), and the Academic Control Scale (Perry et al., 2001) during regular class time. During regular class time, the first author explained to students how to respond to the items, provided definitions of the various emotions that students might experience, had students provide examples of what the various emotions might feel like to ensure they understood the qualifying words on the emotions scale, and then read all items for all questionnaires out loud to students. Then, one day prior to being given the mathematics problem, students were trained to think out loud. The students then heard a practice think-aloud audio file that modeled what not to do followed by an appropriate think out loud example. Finally, students practiced thinking out loud while completing the following mathematics problem, "Kim can walk three kilometers in one hour. How far can she walk in two and a half hours?" Students practiced for approximately 15 minutes.

The following day after think aloud training, students were given the problem to solve (again, during regular class time, which students were told counted toward their grade in mathematics). Students were told that the problem was to be treated as if it were an exam, and were not allowed to work together or copy each other's work during problem solving. As such, students were seated in such a way as to prevent them from cheating (barriers were used, which is normal practice for math tests and for other tests like spelling), and headsets were used to capture their think alouds, with microphones placed close to students' mouths. The decibel level in the room was sufficiently loud that students could not hear one another as they worked on the problem.

Students worked on the problem on consecutive days over three to four days for approximately 1.5 to 2 hours each day (the vast majority of students completed the problem within three days). To ensure all students were thinking out loud, five trained research assistants and the first author were present to prompt students to continue to think out loud if they were silent for more than five

<sup>1</sup> We also conducted three CFAs to assess whether the measurement model resulted in one, two, or three value factors. Our results confirm previous research wherein the one-factor model fit was best with this particular age group.

seconds (a ratio of approximately three to four students per prompter). Each day, students completed the task value and control measures with a specific focus on the mathematics problem they were attempting to solve. At predetermined time intervals as noted above, students then completed the Epistemic Emotions Scale (Pekrun & Meier, 2011) each day. Once students completed the problem (after several days), they submitted their work to the research team. To thank students for their participation, each student received a \$15 iTunes card.

### 3.4. Coding and scoring

#### 3.4.1. Self-regulatory processes

To capture students' self-regulatory processes, a concurrent think aloud protocol was used. Students wore Apple Ear Pods with remote and microphone to capture their voices on digital recording devices. Students' think alouds were then transcribed verbatim by four trained research assistants. Think alouds ranged in length from 90 minutes to 4.5 hours, which resulted in 1086 single-spaced pages of text (29,078 lines). Schoenfeld's (1982), Greene and Azevedo's (2009), and Muis's (2008) think aloud coding schemes and Muis's (2007) theoretical model of self-regulated learning were used as a guide to develop a micro-macro-level coding scheme specifically for mathematics problem solving. To develop the coding scheme, the two longest transcripts were selected. Each transcript was 34 single-spaced pages, for a total of 68 pages. The first author and five research assistants together spent four weeks analyzing the transcripts to identify the micro-level codes, which were then categorized into four macro-level processes based on Muis's (2007) model: task definition, planning and goal setting, enactment, and monitoring and evaluation.

Once these codes were established, the first author then selected three of the longest transcripts from each of the four classes, plus one of the shorter transcripts from each class to ensure comparability across length. The original two used to develop the coding scheme were included in the 16 transcripts chosen, which resulted in a total of 315 single-spaced pages of transcripts. The first author and five research assistants then spent an additional eight weeks working together to establish and modify the coding scheme, coding, and then recoding the transcripts until an acceptable level of inter-rater reliability was achieved. At the end of this process, inter-rater agreement was 85% for the 16 transcripts.

The first author and five research assistants then coded another four transcripts, one from each class, to ensure inter-rater reliability. Total page-length was 80 single-spaced pages. Inter-rater agreement was established at 82%, and disagreements were resolved through discussion. As such, 395 pages (37%) of the transcripts were coded to establish inter-rater reliability. The research assistants then coded the remaining transcripts independently. Following this, frequencies of each of the strategies were examined, and strategies that occurred infrequently were removed from consideration (e.g., averages less than 3 over a 4.5 hour period). Following Greene and Azevedo's (2009) protocol, four macro variables were then created by summing each of the micro variables within that macro code: Phase 2—planning and goal setting, Phase 3—shallow cognitive strategies (e.g., coloring, rereading, calculating), Phase 3—deep cognitive strategies (e.g., summarizing, coordinating information sources, making inferences), and Phase 4—metacognitive strategies (e.g., monitoring, control, evaluation). See Table 1 for examples and definitions of each micro- and macro-level process.

#### 3.4.2. Mathematics achievement

A rubric was developed to score each student's solution to the situational problem. Each element of the problem was given a particular value, and full points were awarded for successfully

completing each element. Partial points were given when aspects were missing, or zero points were given if an element was completely missing or wrong. The total number of points was 50. The first and second author together coded 10 of the solutions to establish consistency in use of the rubric. Agreement was 100%. The two coders then coded 10 additional solutions independently to establish inter-rater agreement. Agreement was 100%. Given high inter-rater agreement, the second author then coded all remaining solutions. See the Appendix for the scoring rubric.

## 4. Results

### 4.1. Preliminary analyses

Skewness and kurtosis values were examined for normality for all variables. For kurtosis, all variables were within an acceptable range (using Tabachnick & Fidell, 2013 criteria of  $<|3|$ ). For skewness, with the exception of plans and goal setting (5.29), variables were within an acceptable range. Given that plans and goals were calculated as an actual frequency with a meaningful zero point, scores were not transformed (see Tabachnick & Fidell, 2013).

Intraclass correlations were also examined for all variables across the two schools. All ICCs were less than .05. As such, nested analyses were not necessary. Collinearity diagnostics were also performed, and results revealed no multicollinearity.

We then examined whether there were gender differences across each of the variables. No gender differences were found for task value, control, or any of the global emotions, epistemic, or activity emotions (all  $p > .10$ ). Gender differences were found, however, for prior knowledge,  $F(1, 76) = 4.61, p < .05, \eta^2 = .06$ , strategy use,  $F(4, 72) = 9.98, p < .001, \eta^2 = .37$ , and achievement,  $F(1, 76) = 9.92, p < .01, \eta^2 = .12$ , with girls scoring higher on all variables compared to boys. As such, prior knowledge was used as a covariate in all subsequent analyses. Moreover, to ensure level of specificity was equivalent across all variables and over time, students' task-specific value and control were used and were averaged across the three days of problem solving (no differences were found across each day for these two constructs). Similarly, for each epistemic and activity emotion, these were also averaged across the three days (again, due to no differences in these emotions across the three days, with the exception of confusion and anxiety, which significantly decreased over time [ $F(2, 94) = 6.88, p < .01, \eta^2 = .13$ , and  $F(2, 94) = 3.52, p < .05, \eta^2 = .07$ , respectively]). For self-regulatory strategies across the three phases of self-regulated learning, total frequency across the three days was used. Means and standard deviations of all variables averaged across the three days are presented in Table 2, and Table 3 presents the zero-order correlations.

### 4.2. Path analysis and mediation model

To test the mediation model presented in Fig. 1, we used Hayes and Preacher's (2013) MEDIANTE SPSS macro, which is recommended with complex models and smaller sample sizes as it maintains higher levels of power while still controlling for Type I errors (see Preacher & Hayes, 2008). Although traditional path analytic approaches suffer from low power with small sample sizes (MacKinnon, Lockwood, Hoffman, West, & Sheets, 2002), this issue can be addressed by using a bootstrapping technique (Hayes & Preacher, 2013). The bootstrapping technique generates a sampling distribution of the effects by pretending the sample is a population, and then draws random resamples of size  $N$  with replacement a very large number of times (e.g., 10,000 times). This resampling of the original sample provides more precise estimates of the effects and yields far more power given the number of times the sample is resampled. The Monte Carlo method (Preacher & Selig, 2012) is then applied to estimate the path coefficients. Using



**Table 1**

Micro- and macro-level definitions, codes, and examples.

| Phase (Macro)/Micro                                  | Code           | Definition  | Examples  |
|--|----------------|---|---|
| <b>Phase 1—Task Definition</b>                       |                |   |   |
| Prior Knowledge Activation                           | PKA            | A learner generates a perception about the task, context, and the self in relation to the task. External and internal conditions play a major role.<br>Searching for or explicitly recalling relevant prior knowledge.  | Prior knowledge activation, beliefs, motivation, and knowledge of strategies are activated during this phase.<br><br>[reading problem] “Your track must have at least one acute angle. I know what that is. It’s an angle that is less than 90 degrees.”<br>[after reading that a reflex angle needs to be included in the diagram] “I know that means an angle that is greater than 180 degrees.”<br>“A reflex angle. That’s more than 180 degrees.”<br>“5 km, which is short for kilometre.”<br>“So that will help us figure out how many people are in each row...”  |
| Identifying Important Information                    | I <sup>3</sup> | Recognizing the usefulness of information.  | e.g., Planning to use means–ends analysis, trying trial and error, identifying which part of the problem to solve first, solving it within a specific amount of time.   |
| <b>Phase 2—Planning and Goal Setting</b>             |                |   |   |
| Making/Restating a Plan                              | P/RP           | The learner begins to devise a plan to solve the problem and sets goals.<br><br>Stating what approach will be taken, what strategy will be used to solve the problem, or what part of the problem will be solved in some sequence. This includes restating plans.   | “First, I have to figure out how many are in each row, then I can figure out how many people fit in each row to fit 120,000 people.”<br>“Let’s just do trial and error.”<br>“So now I’m going to draw it on paper and see if it’s between 4.5 and 5 km.”<br>“So I’m going to start with the track.”<br>“Now I am going to write ‘have to draw rectangle.’”<br>“We have to have an acute angle, obtuse angle and one reflex angle.”<br>“We have to label these angles too.”<br>“I don’t want to spend too much time figuring out the track.”<br>[time goal]<br>“So I need 2, and then 5.”<br>“I want to make sure my calculations are neat.”<br>“Then I have to reflect it twice.” |
| Setting/Restating a Goal                             | G/RG           | A goal is modeled as a multifaceted profile of information, and each standard in the profile is used as a basis to compare the products created when engaged in the activity. This includes restating goals.  |   |
| <b>Phase 3—Enactment</b>                             |                |   |   |
| Hypothesizing  | HYP            | Enactment occurs when the learner begins to work on the task by applying tactics or strategies chosen for the task.<br>Making predictions.  | “The next one is probably going to tell us the information about the design.”<br>[in reference to the learner’s track being large enough] “I think it is going to be enough.”   |
| Summarizing  | SUM            | Summarizing what was just read in the problem statement.  | “Next, the spectator seating area must be divided into sections each section must have seats for 15,000 people. So there, each section has 15,000 people.”<br>“The starting line must be painted with a frieze pattern, this pattern is a rectangular design that has to be, that has been reflected twice, so it has to be reflected twice.”<br>“So you need to draw circles and write down the required information.”   |
| Help Seeking<br>– info<br>– eval                     | HS             | Asking for help from a teacher, peer, or other source.<br>Help seeking for information (info)<br>VERSUS help seeking for evaluation (eval).   | [turns to teacher and asks a question] “But what if my track isn’t exactly 5 km?”<br>“Mrs. [teacher’s name], for the reflex angle would I do it on the outside or the inside?”<br>“So we’re supposed to do something like this?”<br>“What are we supposed to do next?”<br>“Is this correct?”  |
| Coordinating Informational Sources                   | CIS            | Using other sources of information to help solve the problem.   | “Let’s go back to our popplet.” [Popplet includes the concept map, and learner is going back to the concept map he created to help solve the problem].  |
| <b>Phase 3—Enactment continued</b>                   |                |   |   |
| Highlighting/Labeling/Coloring/<br>Drawing/(Writing) | HLC            | Enactment occurs when the learner begins to work on the task by applying tactics or strategies chosen for the task.<br>Highlighting information, labeling information as part of the problem-solving process, or taking notes in reference to the problem.<br>Making a drawing to assist learning or as part of solving the problem | “We can put the starting line just like right there.” [labeling]<br>[you can hear the learner’s pencil] “So its two sides, 2 sides, 3, kind of look like a good drawing [evaluating quality of drawing], 4.”<br>“Like that, like that and like that.”<br>“This is a reflex angle.”<br>“4 C-M.”  |
| Calculating/Measuring                                | CAL            | Solving equations, measuring, or other similar features.  | [adding up the sides] “10 so that’s like 1 km plus 1 km and 400 meters...”<br>“4.4 plus 3.1 plus...equals...”<br>“I’m measuring the starting line.”<br>“I’m just going to re-read this...”  |
| Re-Reading   | R-R            | Re-reading a section of the problem, word for word. Important that it is word for word, otherwise it is summarizing.  |   |

(continued on next page)

**Table 1** (continued)

| Phase (Macro)/Micro  | Code | Definition  | Examples  |
|--|------|---|---|
| Making Inferences  | MI   | Making inferences based on information read or products created from solving the problem.<br>(self-explanation)<br>Explaining why something was done. Key word is "because."  | "So it doesn't say it has to be irregular or regular."<br>"I'm just, I'm multiplying 18 by 6.25 [calculating] because there are 6.25 per white squares." [self-explanation]   |
| Goal-directed search<br><b>Phase 4—Monitoring and Evaluation</b> |      | Various types of reactions and reflections are carried out to evaluate the successes or failures of each phase or products created for the task, or perceptions about the self or context. Reaction and reflection also includes judgments and evaluations of performance on a task as well as the attributions for success or failure. | "I'm looking for another thing that might be useful."<br>Products created are compared to the standards set via metacognitive monitoring. Monitoring and evaluation can include any facet listed above (e.g., progress, motivation, plans, goals, strategies, products like answers or drawings made).  |
| Self-Questioning   | SQ   | Posing a question.  | "But how much is that?"<br>"What is the most important thing?"<br>"So how do we turn meters into km?"   |
| Monitoring   | MON  | Monitoring something relative to goals.   | "I'm not sure there is a reflex angle in my drawing. Let me check."<br>"I might forget that each section must have seats for 15,000 people."<br>[learner is counting the number of sides for the polygon] "So we have 1 side 2 side 3 side 4 side 5 side 6 side 7 sides."<br>"That would be an acute angle, which is kind of hard to draw, this is hard to draw."<br>"I don't really understand this."<br>"I'm not sure."<br>"This is going to be very hard to figure out."<br>"I need help with this one. I don't understand."<br>"So, I don't know."  |
| Judgment of Learning   | JOL  | Learner is aware that something is unknown, not fully understood, or difficult to do.   | Products created are compared to the standards set via metacognitive monitoring. Monitoring and evaluation can include any facet listed above (e.g., progress, motivation, plans, goals, strategies, products like answers or drawings made).   |
| <b>Phase 4—Monitoring and Evaluation continued</b>               |      | Various types of reactions and reflections are carried out to evaluate the successes or failures of each phase or products created for the task, or perceptions about the self or context. Reaction and reflection also includes judgments and evaluations of performance on a task as well as the attributions for success or failure. |   |
| Self-Correcting  | SC   | Correcting one's mistakes.  | "Here are 4 km. Not 4 km. Sorry, 400 meters."<br>"So the first thing was the track had to be has to be a 4 sided [summarizing], not a 4 sided sorry a 7 sided polygon [self-correcting]."<br>"Never mind, I'm not going to put that."<br>"Oops, that was actually an obtuse angle."<br>After counting the number of sides of the polygon, the learner states, "Yes, I have 7 sides. Okay, we're good."<br>"I measured the wrong thing by accident."<br>[after adding up the sides] "3 km, that's way too little."<br>"That's not very neat."<br>[after judging that polygon was not 7-sided] "I'm just going to erase this. It has to be a 7-sided polygon so let's do a different one."<br>"This is difficult."<br>"This is easy." |
| Evaluation   | EVAL | Judging whether goals have been met, whether a particular strategy is working, whether the answer is correct, whether the work is neat, etc. Judgment of all facets that fall under monitoring.   |   |
| Control  | CON  | Changing strategy when monitoring or evaluating results in a determination that goal has not been met.  |   |
| Task Difficulty  | TD   | Statements reflecting the difficulty or easiness of a task.   |   |

this approach, we implemented a moderated mediation analysis to assess our predicted relations. Prior knowledge was included as a covariate for all variables in the model. Control was included as a moderator between value and enjoyment, anxiety, frustration, and boredom. Mediation was tested as a two-step process to assess whether emotions mediated relations between control/value and mathematics achievement, and then whether strategies mediated relations between emotions and mathematics achievement.

The path model with statistically detectable standardized estimates is presented in Fig. 2. We first analyzed whether control moderated relations between value and enjoyment, anxiety, frustration, and boredom. No moderated effects were found. We then modeled all effects as direct or mediated. The total effects model for the first mediation analysis (control and value, emotions, achievement, with strategies and prior knowledge as covariates) was significant,  $F(3, 75) = 11.30$ ,  $p < .001$ ,  $R^2 = .31$ . For our first research

question, whether control and value served as antecedents to epistemic and activity emotions, value was a positive predictor of curiosity ( $B = .39$ ,  $t = 3.25$ ,  $p = .001$ ) and enjoyment ( $B = .53$ ,  $t = 4.91$ ,  $p < .001$ ), and a negative predictor of confusion ( $B = -.30$ ,  $t = -2.57$ ,  $p = .01$ ), anxiety ( $B = -.23$ ,  $t = -2.00$ ,  $p < .05$ ), frustration ( $B = -.40$ ,  $t = -3.45$ ,  $p < .01$ ), and boredom ( $B = -.36$ ,  $t = -2.97$ ,  $p = .004$ ). Control was a negative predictor of confusion ( $B = -.23$ ,  $t = -1.99$ ,  $p < .05$ ), and anxiety ( $B = -.35$ ,  $t = -3.05$ ,  $p = .004$ ). We then examined whether emotions mediated relations between control and value and achievement (our second research question). Results revealed that confusion mediated relations between both value and control and mathematics achievement ( $t = 2.15$ ,  $p < .03$ ,  $t = 2.28$ ,  $p = .02$ , respectively), with point estimates of  $-1.11$  and bias corrected bootstrap confidence intervals (95%) of  $-.08$  to  $-3.19$  for value, and  $-.87$  and bias corrected bootstrap confidence intervals (95%) of  $-.0006$  to  $-3.14$  for control.

**Table 2**

Means and standard deviations for all variables.

|   | Mean   | Standard Deviation |
|---|--------|--------------------|
| Prior Knowledge                           | 86.06% | 12.38              |
| Control <sup>a</sup>                      | 4.06   | .62                |
| Value <sup>a</sup>                        | 3.91   | .70                |
| Surprise <sup>a</sup>                     | 2.63   | 1.16               |
| Enjoyment <sup>a</sup>                    | 3.71   | .95                |
| Curiosity <sup>a</sup>                    | 3.12   | 1.13               |
| Confusion <sup>a</sup>                    | 2.59   | .89                |
| Anxiety <sup>a</sup>                      | 2.00   | .76                |
| Frustration <sup>a</sup>                  | 1.65   | .72                |
| Boredom <sup>a</sup>                      | 1.95   | .84                |
| Planning and Goal Setting <sup>b</sup>    | 50.24  | 23.58              |
| Shallow Cognitive Strategies <sup>b</sup> | 36.23  | 10.44              |
| Deep Cognitive Strategies <sup>b</sup>    | 88.28  | 31.77              |
| Metacognitive Strategies <sup>b</sup>     | 57.41  | 21.19              |
| Mathematics Achievement                   | 84.67% | 11.61              |

Note: Cog = cognitive.

<sup>a</sup> Average based on a Likert scale ranging from 1 to 5.<sup>b</sup> In raw frequency.

For the next analysis (emotions predicting learning strategies predicting achievement, with prior knowledge, control and value as covariates; our third and fourth research questions), bias corrected bootstrap results for the total effects model were significant,  $F(8, 70) = 4.36, p < .002, R^2 = .30$ . For direct effects of emotions on learning strategies, curiosity positively predicted shallow cognitive strategies ( $B = .34, t = 2.46, p = .01$ ) and metacognitive strategies ( $B = .31, t = 2.08, p < .05$ ). Surprise was a negative predictor for planning and setting goals ( $B = -.28, t = -2.00, p < .05$ ), shallow cognitive strategies ( $B = -.39, t = -3.08, p = .003$ ), and deep cognitive strategies ( $B = -.33, t = -2.56, p < .01$ ). Confusion was a negative predictor of shallow cognitive strategies ( $B = -.45, t = -2.93, p < .01$ ) and deep cognitive strategies ( $B = -.28, t = -1.98, p < .01$ ). Interestingly, enjoyment did not predict any processing strategies, whereas frustration positively predicted shallow cognitive strategies ( $B = .33, t = 2.25, p = .02$ ). Anxiety also positively predicted use of shallow cognitive strategies ( $B = .27, t = 2.21, p < .05$ ) as well as metacognitive strategies ( $B = .27, t = 1.98, p < .05$ ). Boredom negatively predicted planning and goal setting ( $B = -.39, t = -2.52, p = .01$ ), deep cognitive strategies ( $B = -.23, t = -1.60, p < .05$ ), and metacognitive strategies ( $B = -.25, t = -1.65, p < .05$ ). Finally, for our last research question, whether learning strategies predicted mathematics achievement, there were three significant positive predictors, which included use of shallow cognitive strategies ( $B = .25, t = 2.01, p < .05$ ), deep cognitive strategies ( $B = .28, t = 2.23, p < .05$ ), and metacognitive strategies ( $B = .29, t = -2.50, p < .01$ ). For mediation, results revealed that shallow cog-

nitive strategies, deep cognitive strategies, and metacognitive strategies mediated relations between both confusion and curiosity and mathematics achievement ( $t = 2.05, p < .05, t = 2.02, p < .05$ , and  $t = 1.99, p < .05$  respectively).

## 5. Discussion

The purpose of this study was to explore the antecedents and consequences of epistemic and activity emotions during complex mathematics problem solving. Pekrun's (2006) control-value theory and philosophical considerations (Morton, 2010) served as the foundations from which to develop specific testable hypotheses. Elementary students were given a complex mathematics problem to solve over a period of several days. Students' perceived control and value for learning mathematics were examined as antecedents to these emotions, and use of planning and goal setting, shallow and deep cognitive strategies, as well as metacognitive strategies were examined as possible consequences to these emotions. With the exception of surprise, we predicted that both control and value would relate to students' emotions during problem solving. We also posited that higher levels of curiosity, enjoyment, and confusion would positively predict use of deep cognitive and metacognitive strategies, whereas surprise, anxiety, and frustration would positively predict shallow cognitive strategies and negatively predict deep cognitive and metacognitive strategies. We also hypothesized that boredom would negatively predict use of all learning strategies. Finally, we predicted that deep cognitive and metacognitive strategies would result in higher levels of problem solving achievement, and would mediate relations between emotions and achievement.

### 5.1. Antecedents

For our first research question, whether perceived control and value serve as antecedents to students' epistemic and activity emotions, results from path analyses revealed support for the majority of the hypothesized relations. As predicted, value was an important antecedent to curiosity and enjoyment, wherein the more students valued mathematics the more curiosity and enjoyment they experienced during problem solving. Conversely, the more students valued mathematics, the less likely they were to experience confusion, frustration, anxiety, and boredom. For perceived control, the more students felt in control of their learning and learning outcomes, the less likely they experienced confusion and anxiety.

These results provide support for Pekrun's (2006) control-value theory of achievement emotions, and are consistent with previous research on the antecedents of students' enjoyment (Buff,

**Table 3**

Correlations for all variables.

|                     | 2     | 3    | 4     | 5     | 6      | 7     | 8      | 9      | 10    | 11     | 12    | 13    | 14   | 15    |
|---------------------|-------|------|-------|-------|--------|-------|--------|--------|-------|--------|-------|-------|------|-------|
| 1. Control          | .46** | -.15 | .21   | .19   | -.38   | -.22* | -.20   | -.15   | .29*  | .33*   | .24*  | .29*  | .14  | .31** |
| 2. Value            |       | -.02 | .39** | .48** | -.41** | -.24* | -.38** | -.35** | .04   | .13    | .15   | .03   | .10  | .18   |
| 3. Surprise         |       |      | .37** | .38** | .08    | .33** | .02    | -.27*  | -.14  | -.20   | -.18  | .03   | -.09 | -.23* |
| 4. Curiosity        |       |      |       | .56** | -.16   | .18   | -.09   | -.34** | .06   | .27*   | .21   | .27*  | .10  | .22*  |
| 5. Enjoyment        |       |      |       |       | -.36** | -.17  | -.42** | -.64** | .07   | .12    | .15   | .04   | .13  | .24*  |
| 6. Confusion        |       |      |       |       |        | .31** | .51**  | .47**  | .03   | -.34** | -.26* | .06   | -.03 | -.25* |
| 7. Anxiety          |       |      |       |       |        |       | .41**  | .15    | .08   | .28*   | .30*  | .27*  | .08  | .36** |
| 8. Frustration      |       |      |       |       |        |       |        | .30*   | .10   | .27*   | .30*  | .15   | .05  | .21   |
| 9. Boredom          |       |      |       |       |        |       |        |        | -.23* | -.19   | -.23* | -.24* | .09  | .14   |
| 10. Plans/Goals     |       |      |       |       |        |       |        |        |       | .54**  | .46** | .71** | .16  | .22*  |
| 11. Shallow Cog     |       |      |       |       |        |       |        |        |       |        | .36** | .47** | .23* | .29*  |
| 12. Deep Cog        |       |      |       |       |        |       |        |        |       |        |       | .49*  | .24* | .35** |
| 13. Metacognitive   |       |      |       |       |        |       |        |        |       |        |       |       | .15  | .27*  |
| 14. Prior Knowledge |       |      |       |       |        |       |        |        |       |        |       |       |      | .47** |
| 15. Achievement     |       |      |       |       |        |       |        |        |       |        |       |       |      |       |

\*  $p = .05$ .\*\*  $p < .01$ .

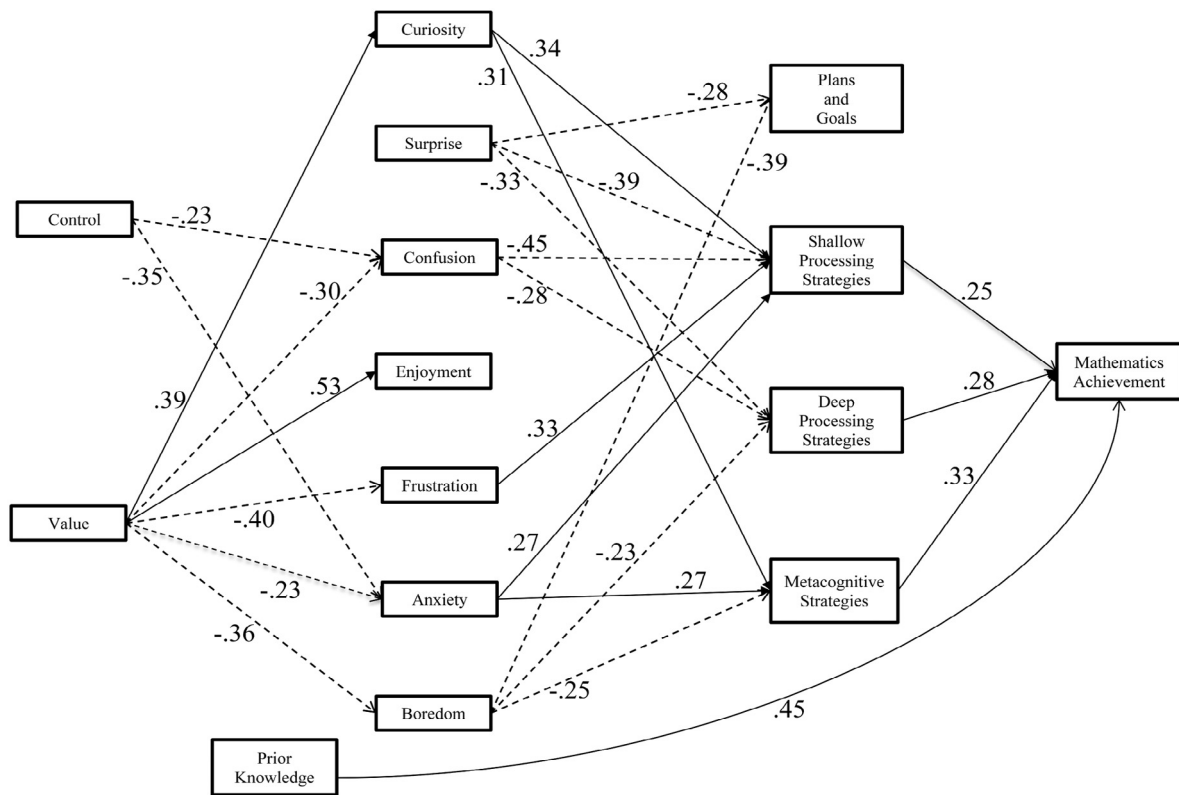


Fig. 2. Final path model.

2014; Buff, Reusser, Rakoczy, & Pauli, 2011; Pekrun, 2000), frustration, anxiety and boredom in mathematics (Dettmers et al., 2011; Frenzel, Pekrun, & Goetz, 2007). More importantly, our results provide initial empirical evidence with regard to plausible antecedents to the epistemic emotions that individuals experience during complex learning. Although philosophers have contemplated the role that importance (value) has on driving curiosity, our research provides evidence that value is a noteworthy antecedent. If students view mathematics as an important educational endeavor, they will more likely be curious about deriving a correct solution to a complex mathematics problem. As we elaborate below, this has important implications for learning when difficulties arise during problem solving. Interestingly, perceived control was a negative predictor of confusion and anxiety such that the more students felt they were in control of their learning and learning outcomes in mathematics, the less likely they were to experience confusion and anxiety during problem solving, which is consistent with previous research on anxiety and its antecedents (Frenzel et al., 2007).

## 5.2. Consequences

Our second research question addressed the consequences of epistemic and activity emotions during complex problem solving, and whether emotions were predictive of multiple phases of self-regulated learning. Results from our research suggest that emotions do predict processes carried out across multiple phases of self-regulated learning, including planning and goal setting, enactment, and evaluation. As such, our work extends theoretical considerations of the role that emotions play in self-regulated learning. Specifically, previous research on emotions focused solely on the enactment phase of self-regulated learning and did not take into consideration how emotions might relate to the plans and goals that individuals set for learning. Given that planning and goal setting is

a key phase of self-regulated learning, particularly for mathematics problem solving (Muis, 2008; Schoenfeld, 1985), results from our study suggest this relationship cannot be ignored. Interventions designed to foster positive emotional experiences should consider the emotions that students initially experience during the task definition phase, and any changes made to task definitions as individuals progress on a task.

Additionally, results from our study also suggest that interventions need to be developed to foster curiosity and to equip students with the necessary skills to address impasses when they occur during complex learning. Specifically, with regard to the role that confusion plays in learning, although previous research with adult populations has shown that confusion can be beneficial for learning when appropriate learning strategies are adopted to resolve that confusion (D'Mello et al., 2014), we questioned whether this was also the case for younger populations who may not have the learning skills necessary to resolve that confusion (Butler & Winne, 1995; Zimmerman & Martinez-Pons, 1990). As such, we explored relations between epistemic/activity emotions and use of shallow cognitive strategies and deep cognitive and metacognitive strategies.

For confusion, results from our study support the concern that elementary students (at least our sample) do not have the necessary skills to resolve confusion when it arises during complex problem solving. Counter to previous research with adult samples (Craig et al., 2004; D'Mello & Graesser, 2011; D'Mello et al., 2014; Graesser et al., 2007; Muis et al., accepted), confusion negatively predicted use of shallow and deep cognitive strategies, and was unrelated to deep metacognitive strategies. Accordingly, when confusion did arise students did not increase their use of metacognitive strategies to reduce confusion. Rather, it appears that confusion in this context behaved more like boredom in that students reduced processing strategies altogether. This is consistent with D'Mello and



Graesser's (2012) results. They found that when confusion persisted and resolution was not achieved after a few attempts, students disengaged, and frustration and boredom ensued. Similarly, VanLehn et al. (2003) found that learners in their study acquired a physics principle in only half of the challenges. They argued that students failed to learn the other principles because they were not able to resolve the impasses. As D'Mello et al. (2014) suggest, it may be worthwhile to distinguish productive confusion from unproductive confusion.

However, drawing from the mathematics education literature, it could very well be that students who experienced confusion during this complex and ill-structured problem may have benefitted from that struggle when given a subsequent well-structured mathematics problem (see Kapur, 2008). That is, as research on productive struggle has found, when students struggle with an ill-structured problem initially, but then are provided support to help solve the problem or are given a subsequent well-structured problem, students outperform others who are not provided scaffolding or given an initial ill-structured problem. As such, future research should explore whether confusion during one complex problem-solving episode has implications for subsequent well-structured problem-solving episodes.

In contrast to confusion, consistent with our predictions and previous research (Muis et al., *accepted*), curiosity was a positive predictor of metacognitive strategies, but also predicted use of shallow cognitive strategies. We interpret these results to suggest that, in the face of a challenging task, curious individuals will more likely engage in monitoring and evaluation of their approaches to solving the problem, and change courses of action when resolutions to issues are not immediately achieved. That is, curiosity fosters better self-regulated learning. Counter to our predictions, however, curiosity also predicted shallow cognitive strategies. To explain this result, shallow cognitive strategies included behaviors such as re-reading the problem statement, calculating distances, or coloring the spectator seating areas. Perhaps because of the complexity of the problem, curious students, who are concerned about acquiring the correct answer, wanted to be meticulous and ensure all aspects of the problem were addressed (Morton, 2010). As such, shallow cognitive strategies may have also been beneficial to achieve this, particularly in the context of mathematics problem solving, which requires these kinds of processing strategies like basic calculations (Schoenfeld, 1985).

Interestingly, students' experiences of surprise lead to a reduction in planning and goal setting, as well as shallow and deep cognitive strategies. Given these results, which are consistent with previous research (Muis et al., *accepted*), it is likely the case that surprise more often led to confusion rather than curiosity for our sample of students. The decrease in these strategies when surprise occurred paralleled relations between confusion and processing strategies. However, like confusion, it may be beneficial for future research to distinguish between surprise that leads to curiosity versus surprise that leads to confusion. Unfortunately, we did not measure emotions dynamically (e.g., as they occurred). As such, we recommend that future research explore dynamic relations between emotions and learning strategies. If surprise occurs, what emotion is likely to arise in the sequence? Do perceptions of value and control predict the likelihood of one emotion over another in these dynamic sequences? We also recommend that future research explore under what context these sequences of emotions occur. For example, what role does prior knowledge play in the activation of these various emotions? Are more knowledgeable students more likely to experience curiosity and less confusion compared to less knowledgeable students? Additionally, we did not measure reciprocal relations between emotions and self-regulatory processes. It may be the case that lack of planning and goal setting predicted higher levels of surprise, rather than the other way around. As such, future work is

needed that explores possible reciprocal relations between emotions and self-regulatory strategies.

To our own surprise, enjoyment was not a significant predictor of any of the processing strategies, despite being the most reported emotion (followed by curiosity). We have no theoretical explanation for the lack of a relationship between enjoyment and any of the processing strategies. We speculate that perhaps the enjoyment that students experienced was a function of the novelty of being part of a research study. Although students reported enjoyment during the activity itself, the object of that emotion may not have been targeted at the process of problem solving but rather because of the researchers circulating the room and providing additional attention that they normally would not experience with only one teacher. Future research is necessary to clarify this lack of relationship.

Consistent with predictions, frustration was a positive predictor of shallow cognitive strategies. For frustration, its positive relationship to shallow cognitive strategies is consistent with Pekrun's (2006) control-value theory and research that supports it (see Pekrun & Stephens, 2012). Similarly, anxiety was a positive predictor of shallow cognitive strategies, but also of metacognitive strategies. As Pekrun et al. (2011) suggest, anxiety can undermine intrinsic motivation but can induce strong extrinsic motivation to invest effort to avoid failure. That is, students may be driven by the fear of failure given the high value they place on the activity and, when this occurs, implement strategies that will help them to succeed in problem solving. As such, students may have invested more effort in metacognitive strategies to ensure they correctly solved the problem, especially given the authentic nature of the task.

Additionally, consistent with our predictions, previous research (Acee & Weinstein, 2010; Pekrun et al., 2010, 2011) and Pekrun's (2006) control-value theory, boredom was a negative predictor of planning and goal setting, deep processing strategies as well as metacognitive strategies. As a negative deactivating emotion, boredom is clearly detrimental to learning, particularly when the learning activity is complex. If the complexity of the task drives students to boredom, they will less likely succeed in solving the problem. Given that deep cognitive and metacognitive strategies were positive predictors of students' problem solving achievement, it is imperative to design learning environments that reduce boredom or that can provide scaffolds for students when boredom arises to shift their learning strategies or regulate that boredom. As previous research in mathematics problem solving has shown, central to successful problem solving is employment of these kinds of deep cognitive and metacognitive strategies (Jacobse & Harskamp, 2012; Muis, 2008; Schoenfeld, 1985). Our results provide support for this.

### 5.3. Educational implications

Results from our study have important educational implications. First, given relations between control and value and the various epistemic and activity emotions, we recommend that teachers relay messages to students about the importance (value) of mathematics, and establish learning environments wherein students' perceived control is heightened. When students perceive mathematics as an important and useful endeavor, they are more likely to experience positive activating emotions and less likely to experience negative ones. Explicit messages that teachers convey can have powerful effects on students' beliefs (Muis & Foy, 2010). Additionally, students should be given meaningful and authentic problems to solve (Windschitl, 2002). By linking what they are learning to why it is important in the real-world context, students' beliefs about the value of mathematics may increase (Muis, 2004).

Finally, given the negative effects that confusion had in our study, it is imperative for teachers to relay the message to students that confusion is a normal emotion to experience. Students also need

to be taught explicit strategies that help resolve confusion when it arises, and teacher support and modeling is critical to help foster these strategies (Zimmerman, 2000). In our study, few students engaged in help-seeking behaviors likely due to the exam-like nature of the task. Even though students were told to ask for help if needed, they rarely took the opportunity to seek guidance during problem solving. Future research is needed to delineate precisely what students were confused about. Perhaps scaffolding during these critical moments would result in productive confusion and students would persist in the face of challenge (Kapur & Bielaczyc, 2012). As such, students should be encouraged to seek help when needed, coupled with an important message that help seeking is not a sign of weakness. Although challenging for teachers, providing the right amount of scaffolding for students and fading that scaffolding over time is critical for the development of students' self-regulated learning (Zimmerman, 2000).

5.4. Conclusion

In conclusion, our research adds to the current literature on emotions and self-regulated learning. To our knowledge, our study is one of the first to explore both the antecedents and consequences of epistemic emotions during complex learning. Second, we broadened research on achievement emotions and their link to self-regulated learning by taking into consideration how emotions influence processes across three of the four phases of self-regulated learning. That is, as both Muis (2007) and Pekrun (2006) proposed, emotions are activated during the task definition phase of learning. Activation of these emotions may then influence planning and goal setting (the second phase of self-regulated learning), enactment of learning strategies (the third phase) as well as evaluative processes like monitoring, evaluation, and control of learning that occur during the fourth phase of self-regulated learning. Our results provide support for Muis's (2007) model, and have important implications for other models of self-regulated learning. Clearly, emotions are important to consider not only in terms of learning outcomes but also with regard to how they foster or hinder self-regulatory processes. Future research we plan will specifically target how to scaffold emotions in ways that foster better learning outcomes.

Our research is also unique in that it was carried out in an authentic classroom situation, and measured students' learning strategies as they occurred in real time. Given that much of the previous work in this area has relied on self-report measures of strategy use, our study also extends that work by incorporating trace data of students' actual learning strategies. To push the field forward, we recommend that future work measure emotions dynamically as they occur. Coupled with traces of learning strategies, researchers will be better equipped to assess whether certain emotions trigger specific strategies, how quickly students react to those emotions, and whether there are non-linear relations that need to be taken into consideration. We also recommend that future research take into consideration different contexts within which students could potentially solve these complex problems. For example, how might emotional experiences differ when students solve these problems in groups? What epistemic, activity, and social emotions might arise, and how might these emotions relate to co-regulated learning? We believe this will be a fruitful line of inquiry that we plan to explore in our future endeavors. Clearly, much more work is necessary before our curiosity is satisfied with regard to the nature of epistemic emotions and their role in complex learning tasks. Confusion may very well be a productive emotion, but students need to have the necessary skills to overcome that confusion in positive ways. Given that these skills can be modeled and taught to young students (MacArthur, 2011; Zimmerman & Labuhn, 2011), we believe there are promising avenues for future intervention research.

Appendix

|  | Your Mark | Total Mark |
|--|-----------|------------|
| Racetrack design:  |           |            |
| 7-sided polygon  |           | 7          |
| perimeter between 4.5 km and 5 km  |           | 4          |
| measures of each line segment (with ruler and label)                             |           | 4          |
| 1 acute angle, 1 obtuse angle, and reflex (180°–360°) [have and label]           |           | 6          |
| Identifies the starting line with an “S”   |           | 1          |
| Spectator area:  |           |            |
| 8 sections   |           | 1          |
| letter identification for each section   |           | 1          |
| 15 squares per section   |           | 1          |
| Starting line frieze pattern:  |           |            |
| rectangular design measuring 6 squares by 3 squares, reflected twice             |           | 3          |
| 1/3 white and 2/3 black  |           | 3          |
| Starting line painting:  |           |            |
| costs 112.50 \$  |           | 1          |
| Calculations:  |           |            |
| 50 cm represents 5000 m  |           | 3          |
| 6 cm + 5 cm + 4 cm + 9 cm + 5 cm + 10 cm + 10 cm = 49 cm                         |           | 3          |
| 15 000 ÷ 1000 = 15 squares   |           | 3          |
| 120 000 ÷ 15 000 = 8 sections  |           | 3          |
| (18 m × 3 m = 54 m <sup>2</sup> ) (1/3 × 54 m <sup>2</sup> = 18 m <sup>2</sup> ) |           | 3          |
| or 1/3 white squares, 6 squares white, 18 white squares total                    |           |            |
| 6.25 \$/m <sup>2</sup> × 18 m <sup>2</sup> = 112.50 \$                           |           | 3          |
| Total:   |           | 50         |
| Percent:   |           |            |

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Dear Student,

Congratulations! You have been selected to join the Start Your Engines Construction Company, which has been hired to build a new Formula One racetrack.

Formula One, often called F1, is the highest class of open-wheel auto racing. The F1 world championship season consists of a series of races, known as the Grand Prix races, most of which are held on specially built circuits.

Here is the Job you are asked to do:

- Design the new racetrack, including a starting line.
- Plan the spectator area.
- Calculate the cost of painting the starting line.

You must keep in mind the specific criteria set by the Formula One Racing Program. These criteria are provided on the next page of this application.

On behalf of the Start Your Engines Construction Company, welcome to the team!  
Mark Getsetgo (Construction Manager)

*Start Your Engines*



The "Start Your Engines" math problem (criteria set by the Formula One Racing Program) will be on the next page of the application. Read the "Start Your Engine" math problem first. When you read the problem, you can highlight the information in the problem using the legend below. The information you highlight can be used to construct the concept map that you will be asked to create after you read the problem.

When you highlight only one word can be chosen at a time, so if you want a few words grouped together click on the words one at a time. For example, if you wanted to highlight "chosen at a time", you would click on "chosen", then click on "at", then click on "a", then click on "time". Since these words are beside each other they would be grouped together as "chosen at a time". The information that you highlight also be found on the next page. The problem is grouped into 4 sections, and when you highlight information from each section, it will be grouped together under its specific section. This is the information you will use to create your concept map. You can use the audio-recording feature to record your thoughts as you read the problem, and play it back to yourself at any time. You can also use the calculator feature to create concept bubbles with equations.

After you read and highlight the problem, create a concept map of the problem using the legend below. This concept map is very important! Once you have created your concept map, click on your "student avatar" and your "student avatar": will study your concept map and use it to learn how to solve the problem. Once the "student avatar" has studied the concept map, click on Take Quiz! and a "teacher avatar" will give the student a quiz. If the student gets a question wrong, that means that you are missing information in your concept map, or you have incorrectly categorized the information. You have only two tries per question, so don't rush and do you best! If the student gets a question right, that means you have correctly categorized the information.

The following is the concept map legend:

**Blue border** represents the title  
**Red border** represents the first question  
**Green border** represents the calculations  
**Black border** represents important information



\*Please note that the symbols in the top left corner will allow you to switch pages.

Start





## Start Your Engines

### Racetrack Design

- ❖ The outline of the track must be drawn inside the central rectangle of your plan.
  - Your track has to be a 7-sided polygon.
  - The length of the outside perimeter of your track must measure between 4.5km and 5km. The measure of each line segment must be clearly indicated.
  - Your track has to have at least 1 acute angle, 1 obtuse angle, and 1 angle that measures more than 180 (You must clearly identify the angles by name on the plan.)
  - The starting line on your track should be identified with an "S".
  - The scale for this problem is 1cm = 100m.

### Spectator Area

- ❖ The spectator area surrounds the track. This area must be able to seat 120,000 people.
  - The spectator seating area must be divided into sections. Each section must have seats for 15,000 people. For this spectator seating area, 1 square = 1000 seats.
  - Each section must be clearly outlined in the spectator area of your plan and identified by a letter.

### Starting Line/Frieze Pattern

- ❖ The starting line must be painted with a frieze pattern. This pattern is a rectangular design that has been reflected twice.
  - The rectangular design must be black and white. White must cover 1/3 of the design.

### Cost of Starting Line Paint

- ❖ The company that was hired to paint the white section of the frieze pattern charges \$6.25 per square metre.
  - The starting line measures 18m by 3m.





Racetrack Design

Spectator Area

Starting Line/Frieze Pattern

Cost of Starting Line Paint



$$\frac{120000}{8} = 15000$$

Racetrack Design

Start Your Engines

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4.5km and 5km.

\$6.25

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Spectator Area

square = 1000 seats.

1/3 of the design.

15,000 people.

reflected twice.

Click the button when you are ready for me to take the quiz!

Take Quiz!



Calculator, Zoom In, Zoom Out, and Target icons

Question: 2/15

Attempt: 0/2

Correct: 0/15

Incorrect: 0/15

What must be the total length of the racetrack in km?



You have already answered this question correctly!

Ok



Next Question

Previous Question

Skip Question

Return to Concept Map





Racetrack Design

Spectator Area

Starting Line/Frieze Pattern

Cost of Starting Line Paint



Start Your Engines

Racetrack Design

Cost of Starting Line Paint

$$\frac{120000}{8} = 15000$$

1cm = 100m.

4.5km and 5km.

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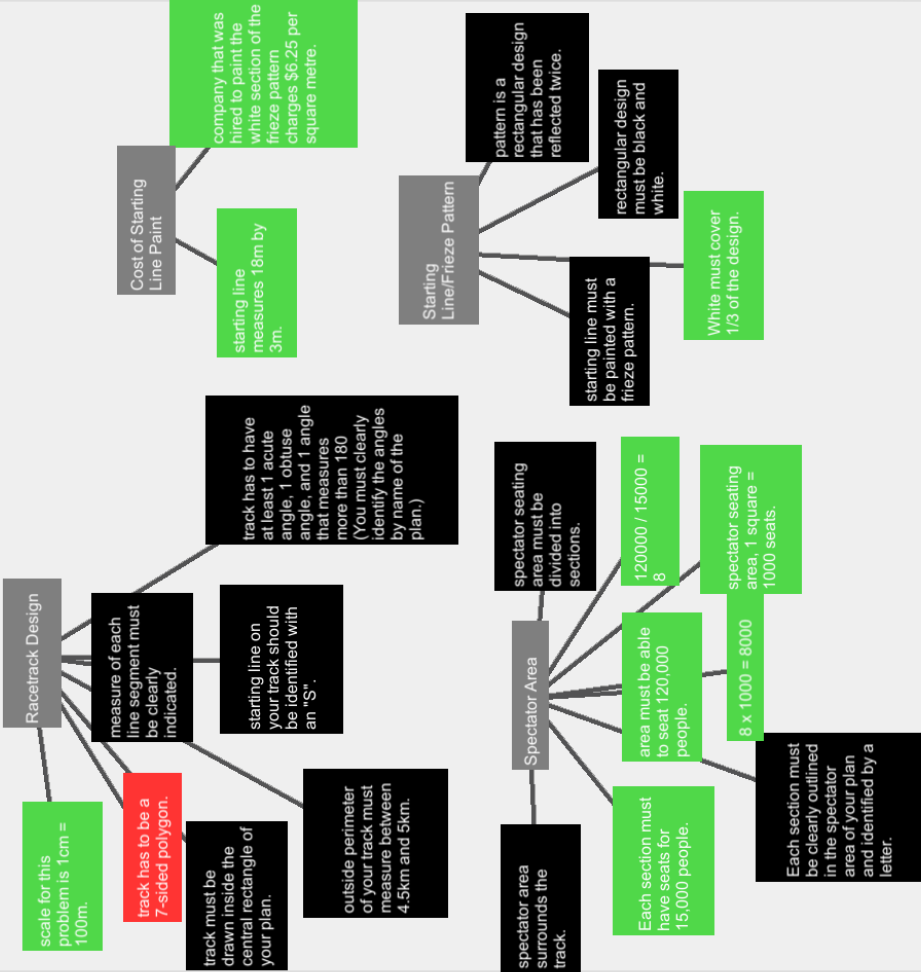
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Start Your Engines





## Start Your Engines

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