



Rapport de recherche

PROGRAMME ACTIONS CONCERTÉES

La réussite en mathématiques au secondaire commence à la maternelle: Synthèse des connaissances sur les pratiques d'enseignement des mathématiques efficaces à la maternelle et au primaire pour réussir l'algèbre du secondaire?

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